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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

AN EMPIRICAL STUDY OF A REFORMULATION OF  
THE CUMULATIVE AVERAGE LEARNING CURVE

by

David George Jenkins

March 1986

Thesis Advisor:

Dan C. Boger

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An Empirical Study of a Reformulation of  
the Cumulative Average Learning Curve

by

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Lieutenant, United States Navy  
B.S., United States Naval Academy, 1978

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

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NAVAL POSTGRADUATE SCHOOL  
March 1986

## ABSTRACT

One aspect of efficient management of resources that cannot be overstated is accurate cost estimation. The learning curve technique used in cost estimation continues to be a significant tool by itself and as an important factor in other cost estimation algorithms. This study conducts an empirical investigation of a theoretical reformulation of the cumulative average learning curve. The model is empirically corroborated by comparison of linear and nonlinear regression results with the classical unit and cumulative average learning curve specifications using two sets of aircraft production data. When autocorrelation was present and subsequently modeled into the data, the resulting linear models were significantly distorted whereas the non-linear models were not. While the model being scrutinized was adequate, the unit learning curve appeared to be the superior model.

Thesis  
1954  
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## I. INTRODUCTION

### A. BACKGROUND

In March of 1972, the General Accounting Office sent a preliminary report to Congress dealing with the acquisition of major weapon systems [Ref. 1:p. 1]. The GAO reported that the Navy had experienced a cost growth of \$19 billion on twenty-four weapon systems in FY 1971, of which 15 percent was attributed to poor cost estimation. Inaccurate cost estimates for weapon systems can result in program delays, cost overruns, acquisition of systems that are not the most cost effective, and a lack of taxpayer confidence in military leaders, to name only a few of the consequences. Congressional concern and a continuing need for better planning estimates have made it imperative that new techniques be developed and old methods be improved to obtain better cost estimates for major weapon system production and acquisition [Ref. 2:p. 1]. In the area of cost estimation, an old technique that continues to be a significant tool is the learning curve.

The first study addressing the learning curve phenomenon was documented by the pioneer of the learning curve, T. P. Wright of the Curtiss-Wright Corporation, in his 1936 paper, "Factors Affecting the Cost of Airplanes" [Ref. 3:p. 32]. Analysis of the data collected for a number of years



beginning in 1922 concerned the relationship of production quantity with cost as measured in direct labor hours. Wright claimed that each time the cumulative production quantity doubled, the average unit cost for that quantity decreased by a constant amount, and that this relationship plotted as a straight line on logarithmic paper. Wright's formulation of the learning curve was:

$$Y_c = aX^b$$

where

X: cumulative production quantity

$Y_c$ : average cost per unit

b: factor of cost variation

a: direct manhour cost for production unit number one

Based on most of the literature available, it can safely be said that the principal factors contributing to the existence of this learning phenomenon include considerably more than just operator learning. Conway and Schultz [Ref. 4:p. 42] believe that learning in aircraft production is influenced by a number of during-production factors including:

- 1) incentive pay
- 2) changes in tooling
- 3) design changes

- 4) management learning
- 5) volume changes
- 6) quality improvements

The rate of a learning curve is usually described by the complement of the reduction achieved when the production quantity is doubled. This value is usually called the slope of the curve and is found:

$$\begin{aligned}
 S &= Y_{2X}/Y_X \\
 &= (2X)^b/X^b \\
 &= 2^b
 \end{aligned}$$

where

b: slope of learning curve

S: fraction to which the cost decreases when production quantity doubles

Wright believed that the cumulative average learning phenomenon plotted linearly on logarithmic scales and the unit learning curve formulation derived from this cumulative equation would be [Ref. 5:p. 266]:

$$\begin{aligned}
 Y_C &= aX^b \\
 Y_T &= Y_C \cdot X \\
 &= aX^{b+1}
 \end{aligned}$$

$$\begin{aligned}\text{So, } Y_X &= a(X^{b+1} - (X - 1)^{b+1}) \\ &= a(b + 1)X^b \quad \text{as } X \rightarrow \infty\end{aligned}$$

where

$Y_C$ : average cost per unit

$Y_T$ : total cumulative cost

$Y_X$ : cost of the Xth unit

a,b: parameters of the formulation

J. R. Crawford, another major contributor to the literature and theory of learning curves, disagreed with T. P. Wright in the log-linear formulation of the cumulative average learning curve [Ref. 6:p. 21]. His disagreement was based on the apparently steep slope between early production units of the unit learning curve derived from the cumulative curve. In Crawford's studies, he described the learning phenomenon in what has been termed the unit learning curve:

$$Y_X = aX^b$$

where

$Y_X$ : cost of the Xth unit

X: cumulative amount of units produced

a: manhour cost for the first production unit

b: factor of cost variation

The cumulative average cost curve derived from the unit curve is [Ref. 6:p. 21]:

$$Y_X = aX^b$$

$$Y_T = a \sum_{X=1}^n X^b$$

$$Y_C = (a \sum_{x=1}^X x^b) / X$$

$$= (a/(1 + b))X^b \quad \text{as } X \rightarrow \infty$$

where

$Y_X$ : cost of the  $X$ th unit

$Y_T$ : total cumulative cost

$Y_C$ : average cost per unit produced

$a, b$ : formulation parameters

For years both the unit learning curve and the cumulative average learning curve have been used almost interchangeably. Womer and Patterson [Ref. 5:p. 266] show and conclude this is so because for large values of  $X$ , each curve is a good approximation for the other. They go on to say that a problem arises, however, since learning curves are generally formulated on the first few units of output to



forecast the cost of an entire production. Even though forecasts may be for large values of  $X$ , the data used to make them are not. Under these circumstances, the estimated cumulative average learning curve, for example, may approach a unit learning curve, but not necessarily the same unit curve that would be approximated from early units. Which log-linear learning curve specification to choose, unit or cumulative, had, through the years, presented a source for inaccurate cost estimation. Although 93 percent of all firms utilize Crawford's unit learning curve [Ref. 7:p. 23], there are sufficient exceptions to the use of this unit curve implying experience seems to be the best method for choosing a particular model.

Following World War II, Gardner Carr of the McDonnell Aircraft Corporation felt learning curves being represented as linear on logarithmic paper was an inaccurate portrayal of the learning phenomenon. In his April 1946 article [Ref. 8:p. 77], Carr felt that the straight line was adequate for overall project statistics but is rarely correct for budget or actual cost finding purposes. He believed that the cumulative average learning curve was S-shaped on the logarithmic scale. Explanations for the various segment shapes of this curve are found in a RAND report by Asher, "Cost Quantity Relationships in the Airframe Industry" [Ref. 6:p. 28].

Another study which suggested that learning curves do not adhere to log-linearity was conducted by the Stanford Research Institute following World War II. The Stanford system utilizes the 'B-factor' which, basically, modifies the standard learning curve for prior experience. The formulation of this learning curve is:

$$Y = a/\sqrt{X + B}$$

where

Y: cost per unit in manhours

a: theoretical first unit cost

X: cumulative quantity produced

B: modification factor

The effect of this formulation is a concave curve on the logarithmic scale. The cost of the first unit is depressed and the curve arcs to the standard learning curve [Ref. 7: p. 8].

Further research that deviated from the log-linearity hypothesis was conducted. Another perspective of the production process is that various departments contribute to the overall quantity of direct labor hours. Generally speaking, these departments are fabrication, subassembly, major, and final assembly. It seems obvious that each department contributing to the learning curve would itself have its own learning curve. In order for the various

departments to have their learning effects sum to an overall production process log-linear learning curve, each of the department slopes must be identical. In practice, the various departments often have different slopes. Summing these curves would result in a departure from log-linearity and arrive at a convex curve whose slope is bounded by the flattest of the component curves. In "Cost Quantity Relationships in the Airframe Industry" [Ref. 6:p. 69], Asher uses this argument while conducting a significant analysis disputing the log-linear hypothesis of the formulation of the learning curve. In his report, he also cites research done previously by P. B. Crouse, G. M. Giannini, and P. Guibert supporting his contentions. Asher concludes, however, that his study

. . . does not discredit the use of the linear progress curve . . . . The linear curve is useful for making extrapolations beyond the data range provided the number of additional units is small. It is clearly a matter of judgement whether or not in a specific instance the linear curve is appropriate . . . . If allowable error is relatively small, a convex curve resulting from predicting each of the component curves separately is probably more appropriate.

Another approach to research in the theory of learning curves has involved the inclusion of production rate as an explanatory variable in learning curve models. In Alchian's 1963 article [Ref. 9:p. 679], he cites work done in 1948 that concluded production rate is not a relevant variable. Whereas as results published by Smith [Ref. 10:p. 138], and

supported by Kinton and Congelton [Ref. 11:p. 92], concluded that production rate plays a significant role in explaining the effects of learning, other studies with contradictory results exist. Womer and Gulledge have produced a considerable literature discussing the effects of production rate which resulted in a final report for the Air Force [Ref. 12: p. 5] addressing the contradictory results of previous research, and they develop a cost function including production rate and the cost-quantity relationship of learning curve theory.

In his article "The Learning Curve: Historical Review and Comprehensive Study" [Ref. 13:p. 302], Yelle states that most of the literature in learning curve theory, from its inception through the 1960's, has focused on primarily military applications in the early years through World War II and on industry and business in the more recent years. Through the years and various paths that research in this area has followed, most of the studies do not reach consistent conclusions. The early goals of developing a general formulation of the learning curve that could be applied to the entire aircraft industry or subsets of it were quickly abandoned. Despite the vast amounts of literature disputing the log-linear relationship between cost and cumulative quantity produced, the unit learning curve is still the most widely used formulation of the learning curve used in cost estimation today [Ref. 7:p. 7].



## B. OBJECTIVES

The preceding pages and references provide a brief summary of the research expended on the theory of the learning curve over the past half century. The important point is the learning phenomenon and the numerous formulations of this theory in aircraft and other industries has been an area of extensive research and continues to be a viable tool in the world of production economics.

The purpose of this research is to conduct an empirical study of still another theoretical reformulation of the learning curve. In "Budgets, Contracts, Incentives and Costs: A Stylized Nexus", by Boger, Jones and Sontheimer [Ref. 14:p. 23], the cumulative average learning curve is reformulated to examine the influence cost forecasting and budget formation have on the incentives bearing on the firm for cost control. The model developed by Boger et. al., a cumulative average learning curve model, and a unit learning curve model will be estimated through simple linear and non-linear regression techniques using several sets of aircraft production data. For each formulation of the learning curve, the models resulting from the two fitting techniques will be analyzed, validated, and compared. Finally, the Boger et. al. model will be compared with the classical learning curve models for empirical validation.

## II. THE MODELS

### A. CUMULATIVE AVERAGE LEARNING CURVE

The cumulative average learning curve, as discussed above, was first formulated by T. P. Wright in the 1930's. The log-linear relationship between cumulative production quantity and average cost per unit is:

$$Y_C = ax^b$$

where

X: cumulative production quantity

$Y_C$ : average cost per unit

b: factor of cost variation

a: direct manhour cost for first unit

The cumulative production quantity is usually expressed as an integer number of units produced. The cost variable is measured in direct manhours expended in the production of the cumulative quantity produced. We expect the learning curve slope, factor of cost variation, to have a negative value when we anticipate the presence of learning in the production of some product. This formulation also presupposes a relatively constant rate of production and uniformity of units produced. Deviations from these last

assumptions are recognizable in a plot of the raw data, i.e., toe up, toe down, bottom out, scallop.

#### B. UNIT LEARNING CURVE

The unit learning curve, as also discussed above, was first formulated by J. R. Crawford. He disagreed with Wright's log-linear formulation of the cumulative average learning curve. Crawford believed the relationship between cumulative quantity produced and the cost of the final unit of that quantity was log-linear and was formulated as:

$$Y_X = aX^b$$

where

- $Y_X$ : cost of the final unit
- $X$ : cumulative quantity produced
- $a$ : direct manhour cost for first unit
- $b$ : factor of cost variation

The same comments and assumptions concerning the cumulative average learning curve apply.

#### C. BOGER, JONES, AND SONTHEIMER MODEL

Boger, Jones, and Sontheimer express the costs of production over a time period as opposed to over the production of cumulative units regardless of time. They use the cumulative average learning curve as the starting point in their formulation.

As discussed above, the typical cumulative average learning curve is of the form:

$$Y(t) = aQ(t)^b \quad (1)$$

where now

$Y(t)$ : average cost per unit

$Q(t)$ : cumulative quantity of units produced through time  $t$

$a, b$ : learning curve parameters

The typical progress function (learning curve) treats the inputs as varying continuously and causing a related continuous variation in some product (output) [Ref. 14: p. 23]. From (1) we can derive an expression for total cost:

$$\begin{aligned} Q(t) \cdot Y(t) &= aQ(t)^b Q(t) \\ X(t) &= aQ(t)^{b+1} \end{aligned} \quad (2)$$

where

$X(t)$ : total quantity of inputs consumed by the production of  $Q(t)$

This specification yields the following marginal requirements,  $dX$ , for an incremented output,  $dQ$ :

$$\frac{dX}{dQ} = a(b + 1)Q^b \quad (3)$$

Now, assume the product emerges in quantities at discrete time intervals. That is, we now develop an algorithm using the cumulative average learning curve formulation based on how many units are produced in a specified time period. In application, we assume that progress or cost per quantity is proportional to productivity achieved in prior production:

$$X_t = \delta_t \frac{X_{t-1}}{q_{t-1}} q_t \quad (4)$$

where

$q_t = dQ$ : amount produced in time period  $t$

$X_t = dX$ : inputs used in time period  $t$

$\delta_t$ : proportionality constant

We assume that learning is derived not only from the preceding period but from all the production prior to the period we are in. So we first set:

$$\frac{X_t}{q_t} = \frac{dX}{dQ} = a(b + 1)Q^b$$

where

$$Q = Q(t)$$

Substituting (4) we get:

$$\begin{aligned} \delta_t \frac{X_{t-1}}{q_{t-1}} q_t / q_t &= a(b + 1)Q^b \\ \delta_t \frac{X_{t-1}}{q_{t-1}} q_t &= a(b + 1)Q^b q_t \end{aligned} \quad (5)$$



We now let  $Q$ , the quantity of units produced up to time  $t$ , be equal to the quantity of units produced through time period  $t-1$ . Now, substituting into (5):

$$\delta_t \frac{X_{t-1}}{q_{t-1}} q_t = a(b+1) \left[ \sum_{j=1}^{t-1} q_j \right]^b q_t \quad (6)$$

Equation (6) assumes learning in period  $t$  is derived only from production in period  $t-1$ . We assume this relationship must hold at previous time periods also. So rewriting (4) and (5) for period  $t-1$ ,

$$X_{t-1} = \delta_{t-1} \frac{X_{t-2}}{q_{t-2}} q_{t-1} = a(b+1) Q^{*b} q_{t-1}$$

where

$Q^*$ : amount of units produced through time period  $t-2$

Therefore,

$$X_{t-1} = a(b+1) \left[ \sum_{j=1}^{t-2} q_j \right]^b q_{t-1}$$

which leads to:

$$\frac{X_{t-1}}{q_{t-1}} = a(b+1) \left[ \sum_{j=1}^{t-2} q_j \right]^b$$

and substituting into (6):

$$\delta_t a(b+1) \left[ \sum_{j=1}^{t-2} q_j \right]^b q_t = a(b+1) \left[ \sum_{j=1}^{t-1} q_j \right]^b q_t$$

$$\delta_t = \left[ \frac{\sum_{j=1}^{t-1} q_j}{\sum_{j=1}^{t-2} q_j} \right]^b \quad \text{for } t = 3, 4, 5, \dots, T \quad (7)$$

Now substituting (7) into (4) we have:

$$x_t = \left[ \frac{\sum_{j=1}^{t-1} q_j}{\sum_{j=1}^{t-2} q_j} \right]^b \frac{x_{t-1}}{q_{t-1}} q_t$$

$$\frac{x_t}{q_t} = \left[ \frac{\sum_{j=1}^{t-1} q_j}{\sum_{j=1}^{t-2} q_j} \right]^b \frac{x_{t-1}}{q_{t-1}}$$

Since this is true for all time periods, we can say:

$$\frac{x_{t-1}}{q_{t-1}} = \left[ \frac{\sum_{j=1}^{t-2} q_j}{t-3} \right]^b \quad \frac{x_{t-2}}{q_{t-2}}$$

$$\frac{x_{t-2}}{q_{t-2}} = \left[ \frac{\sum_{j=1}^{t-3} q_j}{t-4} \right]^b \quad \frac{x_{t-3}}{q_{t-3}} \quad \text{and so on.}$$

So, substituting recursively we have:

$$\frac{x_t}{q_t} = \frac{\left[ \sum_{j=1}^{t-1} q_j \right]^b}{\left[ \sum_{j=1}^{t-2} q_j \right]^b} \frac{\left[ \sum_{j=1}^{t-2} q_j \right]^b}{\left[ \sum_{j=1}^{t-3} q_j \right]^b} \frac{\left[ \sum_{j=1}^{t-3} q_j \right]^b}{\left[ \sum_{j=1}^{t-4} q_j \right]^b} \cdots \frac{\left[ \sum_{j=1}^2 q_j \right]^b}{q_1^b} \frac{x_2}{q_2}$$

$$\frac{x_t}{q_t} = \left[ \frac{\sum_{j=1}^{t-1} q_j}{q_1} \right]^b \quad \frac{x_2}{q_2}$$

$$\frac{x_t}{q_t} = z \left[ \frac{\sum_{j=1}^{t-1} q_j}{q_1} \right]^b$$

where

$z$ : direct manhours per quantity produced in second time period

$t-1$

$\sum_{j=1}^{t-1} q_j$ : total quantity of units produced prior to present time period

$q_1$ : quantity of units produced on time period one

$b$ : factor of cost variation

$X_t/q_t$ : average cost in direct manhours of units produced in time period  $t$

The length of the time period, although it must remain fixed over the data space, can be any length, i.e., day, month, or quarter. The quantity produced in a particular time period need not be an integer amount although partial units produced are generally not found in aircraft production data. As in the cumulative average and unit learning curve formulations, we expect the factor of cost variation to have a negative value. This model also presupposes uniformity between production units and also a constant production rate.

### III. DATA

#### A. GENERAL

The dependent variable in each of the models investigated will involve a cost of some type. In each of our models this cost will be measured as a function of direct manhours expended in the production of some quantity of units. Direct manhours will be defined as those hours spent on fabrication, assembly, production flight, and other production work associated with the basic aircraft. All manhours pertaining to tooling, engineering, planning, testing and subcontracting are not included in this definition. It seems obvious that the way in which direct manhours are accumulated can, and does, lead to inconsistencies due to differences in accounting systems from contractor to contractor. The use of direct manhours has numerous advantages over the use of dollars as a measure of cost. In using direct manhours, we avoid the additional data computations involved in applying price indices to transform all dollar costs into constant dollars. We also avoid inaccuracies in the data caused by using price indices which are inexact figures. Finally, direct manhours is a variable comparable over a group of contractors whereas, due to differences in wage rates from contractor to contractor,



costs measured in dollars are not the best tool for comparison.

The data for this report include aircraft production data for the C-141 and F-102. The C-141 was produced by the Lockheed Corporation and the F-102 was produced by General Dynamics. The C-141 program produced 284 aircraft from July 1962 through April 1968. The C-141 is a large, swept wing, 4 jet engine cargo transport. The data for this study were drawn from Orsini [Ref. 15:p. 104]. Orsini obtained the data from C-141 Financial Management Reports prepared by the contractor, Lockheed Aircraft Corporation, for the Air Force. The C-141 data provided a large sample of data for which a basic model of the aircraft was produced throughout the production program. Uniformity between units produced is a basic assumption in the application of the learning curve theory. Orsini aggregated the monthly production data into quarterly direct manhour production data reducing the total number of data points to twenty-four. Orsini felt this quantity was sufficient for his analysis and the current research is similarly restricted. The data variables used by Orsini and this researcher are:

- 1) direct labor hours per lot per month
- 2) aircraft per lot
- 3) delivery dates of each aircraft

The F-102 program produced 1000 aircraft from 1953 through 1958. The F-102 is a single seat, supersonic, delta

wing, all-weather fighter. The data for this study was drawn from Gulledge and Womer [Ref. 12:p. 73]. A comprehensive cost breakdown by individual airframe was provided by the "F-102 Program Cost History" document--the source of the Womer and Gulledge data. The F-102 program consisted of the production of F-102 airframes and TF-102 airframes. Rather than delete the TF-102 observations for the sake of strict uniformity, these data points were not eliminated since it was assumed that learning was experienced in the production of these airframes. As Womer and Gulledge note, the total manhours expended per airframe can be disaggregated into three parts: details, assemblies, and outside-of-factory labor. Total direct cost per airframe is comprised of only detail and assembly hours. The detail hours are comprised of fabrication hours and assembly hours include subassembly, major assembly, primary assembly, and final assembly hours. After the portion of labor hours expended per airframe outside the factory is deleted, the total direct cost per airframe is left.

## B. REFINEMENT

As already discussed, three models will be utilized in the examination of two sets of aircraft production data. Parameter estimation for these models require the data to be in a particular form for each model. The C-141 production data is available for aircraft grouped into production lots

and the F-102 production data is available for each airframe. Since the models do not each fit the particular form of each data set, adjustments and refinements need to be made to the data to fit the different learning curve formulations.

#### 1. Cumulative Average Learning Curve

The data requirements for the cumulative average learning curve are rather straightforward. The independent variable is the cumulative quantity of aircraft produced. The dependent variable is the average amount of direct labor hours expended per unit in the production of the cumulative quantity produced. The F-102 and C-141 adjusted data used to fit the cumulative average learning curve are tabulated in Appendix A.

The composition of the F-102 data consist basically of total hours expended in the production of each airframe. This data set lends itself to be easily refined to meet the data requirements of the cumulative average learning. As previously discussed, the F-102 total direct manhours per aircraft consisted of three parts: details, assemblies, and outside of factory labor. Table I, extracted from Womer and Gullledge [Ref. 12:p. 86], provided the information necessary to translate the raw data into direct manhours per airframe. Since this table only applied to lots four through eleven, only these 204 observations were utilized. The airframes in lots four through eleven were then ordered with respect to

TABLE I  
PERCENT OF TOTAL MANHOURS ALLOCATED TO  
SPECIFIC ACTIVITIES BY CONTRACT

|                       | Contract |       |       |       |       |
|-----------------------|----------|-------|-------|-------|-------|
|                       | 5942     | 23903 | 29264 | 31174 | 33965 |
| Fabrication           | 19.45    | 21.98 | 21.23 | 16.12 | 18.47 |
| Assembly              | 65.82    | 70.56 | 64.82 | 66.27 | 61.62 |
| Outside of<br>Factory | 14.73    | 7.46  | 13.95 | 17.61 | 19.91 |

delivery sequence number. It was this sequence--1, 2, 3, ..., 204--that provided the independent variable data vector. The sequence of cumulative sums of direct manhours divided by the cumulative amount of airframes delivered for each element of that sequence provided the dependent variable data vector.

The C-141 data were organized into twelve lots. The number of units in each lot and the number of direct man-hours expended in the production of each lot of airframes is provided. The data required for the cumulative average learning curve is arrived at through a series of simple calculations discussed in the RAND Memorandum "An Introduction to Equipment Cost Estimating" [Ref. 16:p. 104]. The cumulative average hours are computed at the final unit in each lot--where the cumulative average hour figures apply. Therefore, twelve data points will be used in the parameter estimation for the C-141 cumulative average learning curve formulation.

## 2. Unit Learning Curve

The data requirements for the unit learning curve are also rather straightforward. The independent variable is the cumulative quantity of aircraft produced. The dependent variable is the amount of direct manhours expended in the production of the final unit of the cumulative quantity produced. The F-102 and C-141 adjusted data used to fit the unit learning curve are tabulated in Appendix B.



The composition of the F-102 data again tends to be easily refined to meet the data requirements of the unit learning curve. Table I is used to translate the raw data of lots four through eleven into direct manhours per airframe. The airframes were then ordered with respect to delivery sequence number. It was this sequence of 204 airframes with each unit's respective direct labor hours required for production that are used as the independent and dependent variable data vectors for the estimation of the parameters of the unit learning curve.

Since the C-141 production data are grouped into lots, a rather gross approximating technique is required to transform the data into the form required by the unit learning curve specification. The average number of labor hours for each lot is treated as if it were an observation on the labor hours required to produce the unit at the lot midpoint. When dealing with a log-linear relationship, the arithmetic midpoint produces unequal areas under the learning curve between the first and last units of each respective lot. The exact determination of a true lot midpoint depends on the lot quantity, type of curve hypothesized, and the true slope of the learning curve [Ref. 16: p. 105]. In order to avoid the shortcomings of the arithmetic midpoint, the algebraic midpoint,  $K$ , discussed in [Ref. 17:p. 44] will be used:

$$K = \left( \frac{m(1 + B)}{(L + .5)(1 + B) - (F - .5)(1 + B)} \right)^{-1/B}$$

m: lot quantity

B: learning curve slope

L: last unit of the lot

F: first unit of the lot

An estimate of B from Womer and Patterson's report [Ref. 5:p. 267], is used in calculating the algebraic midpoint. Again, twelve data points are used in the parameter estimation for the C-141 unit learning curve specifications.

### 3. Boger, Jones, and Sontheimer Model

The data requirements for this model are based on the statement regarding the marginal requirements for incremental outputs of product produced in Boger, Jones, and Sontheimer's paper [Ref. 14:p. 23]. That is, the product emerges in lots or lumps,  $q_t$ , at discrete intervals using discrete inputs,  $X_t$ , of the composite resource (direct labor hours). Therefore, the data requirements for this model are: quantity of units produced each time period and the direct labor hours expended in the production of units produced in each time period.

The complete data base for the F-102 program contains total labor hours for each airframe. This data is not in the form required for the Boger et. al. model. Womer and Gullledge took considerable care in resolving the data

problem in their study [Ref. 12:p. 85]. Their work made the data compatible with the theoretical model they were testing. The information concerning the F-102 program that Womer and Gulledge discuss made it possible to apply some further adjustments to establish a data base compatible with the Boger et. al. model.

As discussed before, the ideal data for the Boger et. al. model is the total number of aircraft produced in a specific time period,  $q_t$ , and the quantity of direct labor hours,  $X_t$ , expended in producing  $q_t$ . Although this data is not directly available, Womer and Gulledge derived the next best alternative--cost by lot per month. Due to non-availability of certain information, Womer and Gulledge only were able to approximate the cost by lot per month for lots four through eleven.

Tables I, II, and III along with the F-102 data base in [Ref. 12:pp. 83-85] provided enough information to adjust the data for lots four through eleven for use in the Boger et. al. model. The first adjustment was to use Table I and the total labor hours expended on each airframe in lots four through eleven to arrive at values for cumulative fabrication and assembly hours for each airframe. As discussed earlier, these hours comprise the direct labor hours expended for each airframe. The next step was to calculate the equivalent airframe units produced per month for each

TABLE II  
F-102 PERCENT DETAIL HOURS COMPLETED

1955

|        | J  | F  | M  | A | M   | J   | J  | A | S  | O  | N  | D  |
|--------|----|----|----|---|-----|-----|----|---|----|----|----|----|
| lot 4  | 20 | 20 | 9  | 9 | 17  | 10  | 13 | 2 |    |    |    |    |
| lot 5  |    | 2  | 15 | 8 | 20  | 15  | 25 | 7 | 4  | 2  | 1  | 1  |
| lot 6  |    |    |    |   | 3.4 | 8.6 | 12 | 5 | 18 | 23 | 17 | 5  |
| lot 7  |    |    |    |   |     |     |    |   | 4  | 4  | 31 | 24 |
| lot 8  |    |    |    |   |     |     |    |   |    |    |    |    |
| lot 9  |    |    |    |   |     |     |    |   |    |    |    |    |
| lot 10 |    |    |    |   |     |     |    |   |    |    |    |    |
| lot 11 |    |    |    |   |     |     |    |   |    |    |    |    |

1956

|        | J  | F  | M  | A  | M  | J  | J  | A  | S  | O  | N  | D |
|--------|----|----|----|----|----|----|----|----|----|----|----|---|
| lot 4  |    |    |    |    |    |    |    |    |    |    |    |   |
| lot 5  |    |    |    |    |    |    |    |    |    |    |    |   |
| lot 6  | 4  | 4  |    |    |    |    |    |    |    |    |    |   |
| lot 7  | 20 | 10 | 5  | 2  |    |    |    |    |    |    |    |   |
| lot 8  | 8  | 25 | 40 | 25 | 2  | 40 | 26 |    |    |    |    |   |
| lot 9  |    |    |    | 8  | 25 | 30 | 40 | 1  |    |    |    |   |
| lot 10 |    |    |    |    | 5  |    |    | 20 | 8  |    |    |   |
| lot 11 |    |    |    |    |    |    |    | 10 | 32 | 48 | 10 |   |

TABLE III

## F-102 PERCENT ASSEMBLY HOURS COMPLETED

|        | J | F | M  | A  | M  | J  | J  | A  | S  | O  | N  | D  |
|--------|---|---|----|----|----|----|----|----|----|----|----|----|
| lot 4  |   | 5 | 15 | 10 | 12 | 23 | 15 | 3  | 2  | 10 | 5  |    |
| lot 5  |   |   |    | 5  | 15 | 15 | 20 | 10 | 15 | 10 | 5  | 3  |
| lot 6  |   |   |    |    |    | 5  | 5  | 15 | 15 | 20 | 15 | 10 |
| lot 7  |   |   |    |    |    |    |    |    |    |    | 10 | 20 |
| lot 8  |   |   |    |    |    |    |    |    |    |    |    |    |
| lot 9  |   |   |    |    |    |    |    |    |    |    |    |    |
| lot 10 |   |   |    |    |    |    |    |    |    |    |    |    |
| lot 11 |   |   |    |    |    |    |    |    |    |    |    |    |

1956

|        | J  | F  | M  | A  | M  | J  | J  | A  | S  | O  | N | D |
|--------|----|----|----|----|----|----|----|----|----|----|---|---|
| lot 4  |    |    |    |    |    |    |    |    |    |    |   |   |
| lot 5  | 2  |    |    |    |    |    |    |    |    |    |   |   |
| lot 6  | 10 | 5  |    |    |    |    |    |    |    |    |   |   |
| lot 7  | 25 | 20 | 15 | 10 |    |    |    |    |    |    |   |   |
| lot 8  | 2  | 25 | 35 | 30 | 7  |    |    |    |    |    |   |   |
| lot 9  |    |    |    | 4  | 20 | 40 | 30 | 6  |    |    |   |   |
| lot 10 |    |    |    |    |    | 20 | 40 | 30 | 10 |    |   |   |
| lot 11 |    |    |    |    |    |    | 5  | 35 | 45 | 15 |   |   |



lot. This was calculated by first determining the empirical production rates for each lot:

$$Y_f = \frac{\sum \text{DMH}_f}{\text{aircraft in lot}} \quad \text{for lots 4, 5, 6, ..., 11}$$

$$Y_a = \frac{\sum \text{DMH}_a}{\text{airframes in lot}} \quad \text{for lots 4, 5, 6, ..., 11}$$

Production rate (fab) =  $1/Y_f$

Production rate (assem) =  $1/Y_a$

$\text{DMH}_f$ : direct manhours for fabrication

$\text{DMH}_a$ : direct manhours for assembly

The production rates for fabrication and assembly were then applied in conjunction with Tables II and III to the cumulative fabrication and assembly hours per month per lot, then added to arrive at equivalent aircraft produced per month per lot. These results were then summed across lots four through eleven for each month appropriately using Tables II and III to arrive at equivalent units produced per month. Direct labor hours expended per month on the equivalent quantity of airframes produced per month was similarly calculated. The adjusted F-102 production data per month for lots four through eleven for use in the Boger et. al. model is summarized in Appendix C.

The original form of the C-141 data made available to Orsini by the Air Force Plant Representative Office was direct manhours per lot per month expended as direct labor hours as defined previously and the quantity of aircraft per lot. Orsini then aggregated this monthly data into quarterly data points and tabulated it as direct manhours per lot per quarter. The adjustments made to the data by Orsini for his analysis were compatible with the refinements required by the Boger et. al. model. Average production rate for each lot was first determined by dividing total aircraft in each lot by the total amount of direct labor hours attributed to the production of each respective lot. This average production rate was then applied to the tabulated quarterly data to arrive at equivalent units produced per lot per quarter. The equivalent units produced per lot per quarter and direct labor labor hours per quarter were then summed across each lot for the quarters each lot was worked on to arrive at equivalent units produced per quarter and direct labor hours expended per quarter. The data, as refined by Orsini, used in the Boger et. al. model is tabulated in Appendix C.

#### IV. METHODOLOGY

##### A. LINEAR REGRESSION

Historically, it has usually been assumed that the relationship between the independent and dependent variables of a learning curve specification is log-linear. This assumption has made it particularly easy to estimate the learning curve parameters through simple linear regression when only one independent variable is used. In this study, the least squares, normal error regression model is utilized. The normal error model is:

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \quad \text{for } i = 1, 2, 3, \dots$$

where

$Y_i$ : observed response of the  $i^{\text{th}}$  trial

$X_i$ : the level of the independent variable in  $i^{\text{th}}$  trial

$\beta_0, \beta_1$ : regression parameters

$\epsilon_i$ : residuals which are distributed  $N(0, \sigma^2)$

Normality of the error terms seems reasonable since the residuals probably represent the accumulation of many effects that are omitted from the model. The cumulative error term,  $\epsilon_i$ , would tend to comply with the central limit theorem and approach normality. Since the error terms are

assumed to be normally distributed, the assumption of no correlation between residuals becomes one of independence. Still yet, the assumption of normality allows one to perform some parametric statistical tests in evaluating the statistical significance of the estimated parameters and the aptness of the model.

## B. NON-LINEAR REGRESSION

Non-linear regression software in STATGRAPHICS [Ref. 18: pp. 19-35] is used as an alternative method of parameter estimation. In this procedure, least squares estimates of the parameters of a non-linear model are determined. The learning curve formulations in this study are inherently non-linear when the data are in their raw form. The non-linear model is:

$$Y_i = aX_i^b + \epsilon_i \quad \text{for } i = 1, 2, 3, \dots$$

where

$Y_i$ : observed response of the  $i^{\text{th}}$  trial

$X_i$ : level of the independent variable of  $i^{\text{th}}$  trial

$a, b$ : regression parameters

$\epsilon_i$ : residuals which are distributed  $N(0, \sigma^2)$

The non-linear regression method utilized in the STATGRAPHICS software was developed by D. W. Marquardt and represents a compromise between the linearization (Taylor series) method and the steepest descent method of non-linear

parameter estimation. Marquardt's compromise has been described as combining the best features of the linearization and steepest descent methods while avoiding their most serious limitations. A detailed discussion and references for this algorithm are contained in Draper and Smith's Applied Regression Analysis, Second Edition [Ref. 19: p. 471]. An important aspect of non-linear regression that deviates from the linear case is worth mentioning. When the error term of the non-linear model is assumed to be normally distributed, the parameter estimates are no longer normally distributed and the sample residual variance is no longer an unbiased estimate of the residual variance. While suitable comparison of mean squares can be made visually, the usual F-tests for regression and lack of fit are not valid, in general, for the non-linear case [Ref. 19:p. 484].

### C. DATA ANALYSIS

Examination of the observed residuals of a regression model is an important aspect of any regression technique. If the model is appropriate, the observed residuals should reflect the properties assumed for the error term in the regression model. In this study, both graphical and statistical tests involving the residuals will be performed. Evaluation of the residuals of the various models to be considered will address possible departures from the model including: the regression model does not hold, the error

terms do not have constant variance, the error terms are not independent, the model fits all but one or a few outliers, and the error terms are not normally distributed.

After fitting a model to the data, residuals falling into a horizontal band centered at zero displaying no systematic tendencies to be positive or negative and appearing to be randomly scattered would suggest the assumptions of the model do not appear to be violated. This would imply the model is well suited to the data. If this is not the case, remedial measures would need to be taken. Generally speaking, there are two types of remedial measures that are normally followed: abandon the model altogether or use some transformation on the data so the model is appropriate for the transformed data. In this report, only two aspects of data transformation will be reckoned with: autocorrelation and the handling of outliers. When these two problems are dealt with and further residual analysis clearly implies the assumptions of the model are not met, the model will be rejected.

#### 1. Autocorrelation

The regression models of ordinary least squares or maximum likelihood techniques consider the stochastic disturbance terms, the residuals of the regression, to be either uncorrelated or independent normal random variables. In the application of regression models to learning curves,



we use time series data. The assumption of no correlation or independence between error terms for time series data is often inappropriate. The observed correlation between residuals of regression modeling is called autocorrelation or serial correlation.

Neter and Wasserman outline the problems associated with autocorrelation:

- i) The regular least squares regression coefficients are still unbiased but no longer have the minimum variance property and may be quite inefficient.
- ii) The mean squared error (MSE) may seriously underestimate the variance of the error terms.
- iii) The estimated standard deviation of the regression coefficients may be seriously underestimated and  $R^2$  may be overestimated.
- iv) The confidence intervals and tests using the student's  $t$  and  $F$  distributions are no longer strictly applicable. [Ref. 20:p. 352]

In this study, the existence of first order autocorrelation, AR [1], will be investigated graphically and will be statistically tested using the Durbin-Watson test. If autocorrelation indeed exists after examination of the residuals, this information will be used to improve the regression model. The autocorrelation will be modeled and accounted for in a transformation of the model data.

The first-order autocorrelation error model discussed by Neter and Wasserman [Ref. 20:p. 353] for a simple linear regression is:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$

$$\epsilon_t = \rho \epsilon_{t-1} + \mu_t$$

where

$\rho$ : autocorrelation parameter,  $|\rho| < 1$

$\mu_t$ : independent and distributed  $N(0, \sigma^2)$

The following discussion also applies in a nonlinear model when the error term is additive. It can be shown that the properties of the error terms lead to the following conclusions:

$$i) \quad E(\epsilon_t) = 0$$

$$ii) \quad \text{var}(\epsilon_t) = \sigma^2 \sum_{s=0}^{\infty} \rho^{2s}$$

$$iii) \quad \text{cov}(\epsilon_t, \epsilon_{t-s}) = \rho^s \left( \frac{\sigma^2}{1 - \rho^2} \right) \quad s \neq 0$$

These imply the error terms for the first-order autoregressive model are autocorrelated unless the autocorrelation parameter,  $\rho$ , equals zero [Ref. 20:p. 357].

When the autocorrelation parameter,  $\rho$ , is not zero, it will be necessary to estimate the value of  $\rho$  for use in the autoregressive structure as a source of additional information in our regression model.

Following a graphical inspection of the residuals, the Durbin-Watson test will be utilized to test the hypothesis:

$H_0: \rho = 0$  implying no autocorrelation

$H_1: \rho > 0$

The test statistic,  $D$ , used in this text is:

$$D = \frac{\sum_{i=2}^n (e_i - e_{i-1})^2}{\sum_{i=1}^n e_i^2}$$

where

$e_i$ :  $i^{\text{th}}$  residual of the regression model

$n$ : number of data points used in the regression

If we reject the null hypothesis, this test-statistic,  $D$ , can be used further to estimate the autocorrelation coefficient,  $\rho$ . The estimate of  $\rho$ ,  $r_1$ , is discussed by Neter and Wasserman [Ref. 20:p. 358] and is:

$$r_1 = \frac{\sum_{i=2}^n e_{i-1}e_i}{\sum_{i=2}^n e_{i-1}^2} \quad (1)$$

For sufficiently large  $n$ , an alternative estimator of  $\rho$  derived by Theil and Nagar [Ref. 21:p. 164] is:

$$r_2 = \frac{1 - \frac{D}{2} + \left(\frac{K}{n}\right)^2}{1 - \left(\frac{K}{n}\right)^2} \quad (2)$$

where K is the number of parameters to be estimated in the regression model.

When  $n \gg k$  then

$$r_3 \approx 1 - \frac{D}{2} \quad (3)$$

The estimator for the autocorrelation parameter,  $\rho$ , in equations (2) and (3) will be used in this study.

The iterative method of incorporating the first-order autoregressive model into the regression model is used and discussed in Neter and Wasserman [Ref. 20:p. 361] and Intriligator [Ref. 21:p. 164]. The data are first transformed:

$$X_i' = \sqrt{\left(1 - r_j^2\right)} X_i \quad \text{for } i = 1; j = 1, 2, \text{ or } 3$$

$$Y_i' = \sqrt{\left(1 - r_j^2\right)} Y_i \quad \text{for } i = 1; j = 1, 2, \text{ or } 3$$

$$X_i' = X_i - (r_j \cdot X_{i-1}) \quad \text{for } i = 2, 3, \dots, n; \\ j = 1, 2, \text{ or } 3$$

$$Y_i' = Y_i - (r_j \cdot Y_{i-1}) \quad \text{for } i = 2, 3, \dots, n; \\ j = 1, 2, \text{ or } 3$$

The regression is then performed with the transformed data. The Durbin-Watson test is then employed to test whether the

new residuals for the transformed data are uncorrelated. The procedure discussed above continues until the Durbin-Watson null hypothesis is accepted.

## 2. Outliers

The presence of outliers can cause some difficulty when fitting a model using the least squares method. Outliers can either be errant observations or perhaps result due to an interaction with a variable that is not included in the model. In either case, when outliers exist, those particular data points should be addressed. If evidence exists that abnormal circumstances surround a particular data point, it is safe to discard it. In order to address outliers, it is obvious that the analyst must be familiar with the data or have the resources to adequately address them. In this report, the resources to adequately address the nature of outliers does not exist; therefore, residuals which lie greater than  $\pm 4 \sqrt{\text{MSE}}$  from zero will be designated as outliers and rejected but annotated.

## 3. Normality of Error Terms

As discussed by Neter and Wasserman [Ref. 20: p. 107], small departures from normality do not create any serious problems in the fitting of the model. Major departures, on the other hand, should be of concern. The normality assumption will be graphically addressed through probability and symmetry plots. A rough statistical test

addressing normality of the error terms is discussed in Neter and Wasserman [Ref. 20:p. 107]. If 90 percent of the standardized residuals,  $e_i / \sqrt{\text{MSE}}$ , fall between the appropriate standard normal values or the corresponding student's t-values for small sample sizes, the normal assumption will not be rejected.

#### 4. Homoscedasticity

The assumption of constant variance of the residuals will also be addressed graphically and statistically. Residual plots will initially be inspected prior to conducting a non-parametric rank correlation test between the absolute value of the residual and the value of the independent variable as discussed in Conover [Ref. 22:p. 255]. The assumptions of constant variance will be rejected if the hypothesis of no correlation is rejected in this non-parametric test.

#### D. INFERENCES CONCERNING PARAMETER ESTIMATION

Following verification of the underlying assumptions of a simple linear regression, it is of interest to investigate the statistical significance of the parameter estimates in the model:

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

It is of interest to initially test the hypothesis:



$$H_0: \beta_1 = 0$$

$$H_1: \beta_1 \neq 0$$

to see if there is a statistically significant linear relationship between the independent and dependent variables. It can be shown that if the underlying assumptions of the model hold, the parameter estimate of  $\beta_1$ ,  $b_1$ , is normally distributed [Ref. 20:p. 53]. Therefore,  $(b_1 - \beta_1)/s(b_1)$  is distributed as  $t(n - 2)$ . Furthermore, the test to decide whether  $\beta_1$  is statistically equal to zero is based on the test statistic:

$$T_1 = b_1/s(b_1)$$

The decision rule, of a significance level  $\alpha$ , is given by Neter and Wasserman as [Ref. 20:p. 61]:

Accept  $H_0$  if  $|T_1| \leq t(1 - \alpha/2, n - 2)$

Otherwise reject  $H_0$

Similarly, it can be shown that inferences concerning  $\beta_0$  are analogous to those for  $\beta_1$  [Ref. 20:p. 61].

The usual tests that are appropriate in the linear model are, in general, not appropriate when the model is non-linear. Draper and Smith [Ref. 19:p. 484] discuss why this

is so and also present a practical procedure that can provide a measure of possible lack of fit for a non-linear model. In the non-linear case, no statistical tests concerning the parameter estimates will be discussed in this study. Instead, the results of the non-linear regression will only be compared to those of the simple linear regression.

#### E. VALIDATION

Since time series data is being used, it is not possible to split the developmental data and the validation data randomly. For each learning curve formulation and the two methods of parameter estimation, roughly, the first seventy-five percent of the data is used to fit each regression model. The remaining data is saved to validate the forecasting ability of the fitted model. While the validation phase of model building is important, the criteria of the validation phase, that is, determining how well a model forecasts, is subjective and goodness can vary depending on the needs of the user. In this research, several measures of forecasting accuracy will be used to quantify model results. The measures selected for this analysis are the mean percent error (MPE), the mean absolute percent error (MAPE) and the Pearson correlation coefficients adjusted for degrees of freedom. MPE is defined as:

$$MPE = \frac{100}{n} \sum_{t=1}^n (A_t - P_t) / A_t$$

MAPE is defined as:

$$MPE = \frac{100}{n} \sum_{t=1}^n |A_t - P_t| / A_t$$

where

$A_t$ : actual or realized value at time  $t$

$P_t$ : prediction of forecast value at time  $t$

The Pearson correlation coefficients are defined as:

$$R^2 \text{ (fitted)} = 1 - \frac{\text{Var}(r)/\text{dof}}{\text{Var}(Y)/\text{dof}}$$

$$R^2 \text{ (Validation)} = 1 - \frac{\text{Var}(rr)/\text{dof}}{\text{Var}(Y)/\text{dof}}$$

where

$\text{Var}(r)$ : sample variance of the residuals of the fitted model

$\text{Var}(rr)$ : sample variance of the residuals of the forecast values

$\text{Var}(Y)$ : sample variance of the developmental dependent data

Whereas MPE provides a measure of the percent bias in the forecasts, MAPE will always be at least as large as MPE and provides a measure of dispersion of the forecasts (see Boger and Jayachandran, Ref. 23:p. 11). Comparison of  $R^2$  (fitted)

and  $R^2$  (validation) quantitatively evaluates the relative variability of the forecasting ability of the model beyond the developmental range.

In this study, the level of the independent variable beyond the developmental range is fixed. The conditional predictions of the dependent variable,  $Y_t/X_t$ , for the regression models for each learning curve specification are based on the following relation:

1) Linear Regression Model

a) Autocorrelation is not modeled

$$\begin{aligned} \text{LN } \hat{Y}_t &= \text{LN } \hat{\beta}_0 + \hat{\beta}_1 \text{ LN } X_t \\ \hat{Y}_t &= \exp(\text{LN } \hat{Y}_t) \end{aligned}$$

b) Autocorrelation is modeled

$$\hat{Y}_t = \hat{\rho} \hat{Y}_{t-1} + (1-\hat{\rho})\hat{\beta}_0 + (X_t - \hat{\rho} X_{t-1})\hat{\beta}_1$$

where  $\hat{Y}_{t-1}$  is equal to  $\exp$  {the last fitted value of the developmental data} for the initial predicted value.

2) Nonlinear Regression Model

a) Autocorrelation is not modeled

$$\hat{Y}_t = \hat{\beta}_0 \hat{X}_t^{\hat{\beta}_1}$$

b) Autocorrelation is modeled

$$\hat{Y} = \hat{\rho} \hat{Y}_{t-1} + \hat{\beta}_0 (\hat{X}_t^{\hat{\beta}_1} - \hat{\rho} \hat{X}_{t-1}^{\hat{\beta}_1})$$

where  $\hat{Y}_{t-1}$  is equal to the last fitted value of the developmental data for the initial predicted value.

where

$\hat{\beta}_0, \hat{\beta}_1$ : estimated parameters of the regression

- $X_t$ : independent variable of the bivariate data that was not used for developing the model
- $\hat{\rho}$ : estimated autocorrelation parameter

## F. COMPARISONS

In this study, three learning curve specifications are being investigated: the unit learning curve, the cumulative average learning curve, and the Boger et. al. learning curve. Each specification will be fitted using both a simple linear regression model and a nonlinear regression model.

### 1. Regression Models

The first comparison that will be investigated, which is of secondary interest in this study, will be the relative fit of each model and the differences between the linear regression and nonlinear regression methods, with and without transformations of the data for autocorrelation, for each learning curve specification. The approach to be used for these comparisons will be strictly graphical. For each model specification the dependent variable of the developmental data will be plotted against the observed dependent variable of the developmental data and each of the fitted variables.

### 2. Learning Curve Specifications

The basis for comparison between the unit, cumulative average, and the Boger et. al. learning curve.

specifications are the differences between actual cost per lot and each model's fitted cost per lot. Each model's fitted cost per lot can be arrived at through some relatively simple calculations using the data refinement procedures discussed above, applied to the results of each regression technique. The initial comparison of the fitted lot costs will be done graphically. For each model specification and regression technique, the observed cost per lot and the fitted cost per lot will be plotted against the respective lot numbers for the data within the developmental range. Where the difference between observed and fitted lot costs are not obviously different by graphical means, a statistical test will be employed to attach statistical significance to the difference. The non-parametric test to be utilized will be the Kruskal-Wallis [Ref. 22:p. 229] where the populations are the different model specifications and regression techniques. The samples within each population are the absolute values of the differences between the observed and fitted cost per lot.



## V. RESULTS

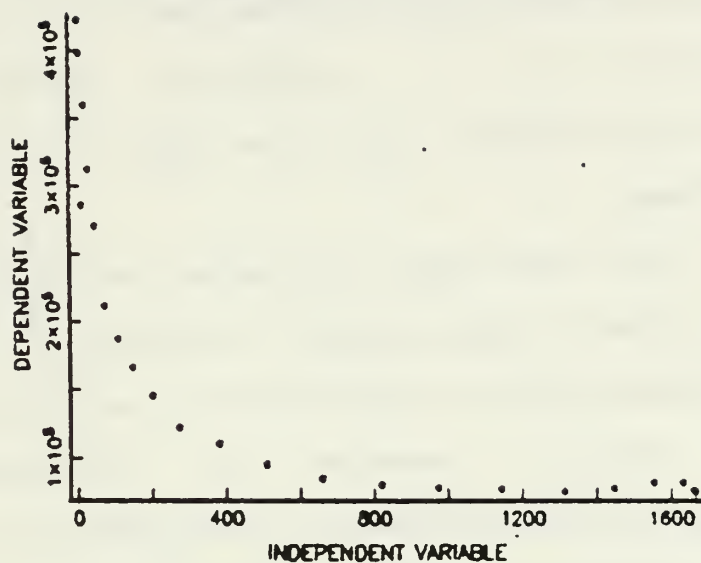
### A. DATA ANALYSIS

Two sets of production data and three learning curve specifications for each data set were investigated in this research. A fairly extensive analysis was performed on the residuals of each type of regression for each learning curve specification and each data set. The results of each analysis, generally, led to further modifications of the data calling for even more regressions and residual analyses. Twenty-six regressions, sixteen linear regressions and ten nonlinear regressions, were performed during the course of this study. For the sake of brevity, only one analysis for a single learning curve specification and production data set, which was typical of the analyses performed in all other cases, will be discussed at length. The results of the other regressions and analyses are tabulated in Appendices D, E, F, G, H, and I.

#### 1. Boger et. al. Model: C-141 Data Analysis

The first 18 of the 24 total bivariate observations were selected to fit the linear regression model for the Boger et. al. specification of the learning curve. The remaining six data points were withheld for validation purposes. Figure 1 is a scatter plot of the raw data and

# RAW DATA: 18 OBSERVATIONS



# LN TRANSFORMED DATA: 17 OBSERVATIONS



Figure 1. Raw Data and Ln Transformed Data

the natural log ( $\ln$ ) transformed data. The  $\ln$  transformed data scatter plot has seventeen data points since the first observation of the independent variable vector was necessarily omitted since its value is infinity when  $\ln$  transformed.

The first linear regression was performed using the 17 data points (observations 2 through 18). Inspection of the residuals plotted against time and against the fitted values, Figure 2, revealed that the residuals were not patternless. The systematic structure of the residuals implied that the residuals did not reflect the assumptions of the linear model. The cyclic pattern of the residuals, furthermore, suggested the presence of first-order autocorrelation and encouraged more investigation. The Durbin-Watson statistic derived from this set of residuals led to a rejection of the null hypothesis ( $H_0: \rho=0$ ) implying statistical significance of the presence of first-order autocorrelation. The initial inspection of the residuals also addressed the question of outliers. Since no residuals were outside the interval specified for data rejection, no observations were omitted from the data set. Table IV highlights the results of the initial linear regression.

Since the sample size was small in relation to the number of parameters being estimated, the Theil and Nagar estimate for the first-order autocorrelation,  $r_2$ , was utilized. The values in parentheses adjacent to the

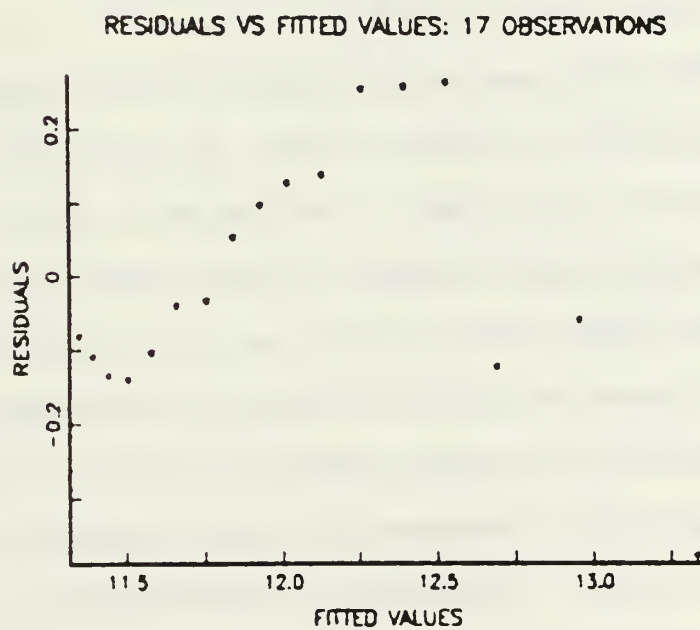
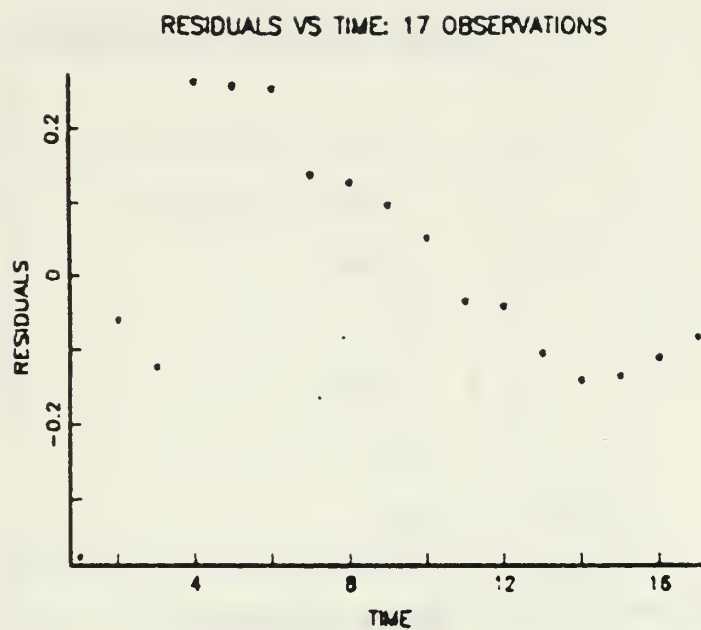


Figure 2. Residual Plots

TABLE IV  
LINEAR REGRESSION 1 RESULTS

$\ln \hat{\beta}_0$  : 13.332 (123.21,  $\alpha < .001$ )  
 $\hat{\beta}_1$  : -.2821 (-13.00,  $\alpha < .001$ )  
 D.W. : .5941  
 N : 17  
 $\hat{\rho}$  : .6987  
 $R^2$  : .92  
 $R^2$  adj. : .91

estimated parameters are the student's t statistics for the respective coefficients.

The autocorrelation was then modeled into the  $\ln$  transformed data resulting in Figure 3. The data point in the upper left hand corner seems to be a typical result when autocorrelation is modeled into the data using the technique employed in this study. A second linear regression was performed on these 17 observations. The scatter plot of the residuals plotted against time and against the fitted values, Figure 4, again, was not patternless and suggested the presence of autocorrelation. Due to the small sample size, the first observation had a dramatic effect on the regression and, subsequently, the residuals. The Durbin-Watson statistic again reflected a statistically significant amount of autocorrelation present in the residuals. Further

# AUTOCORRELATION MODELED: 17 OBSERVATIONS

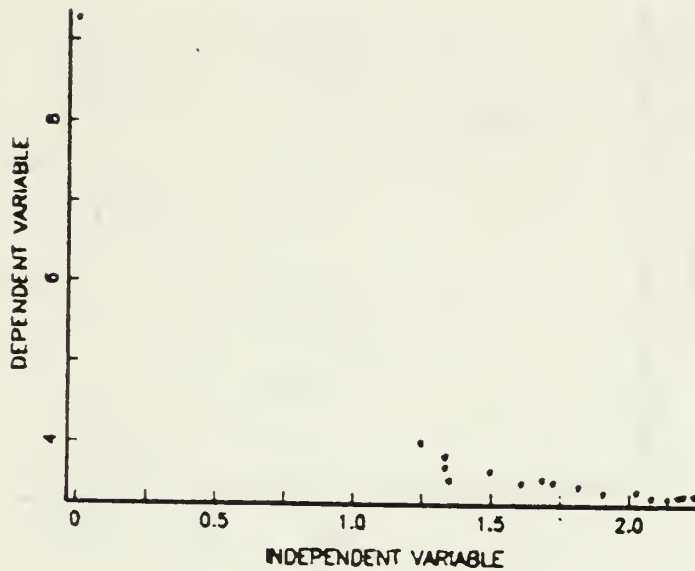


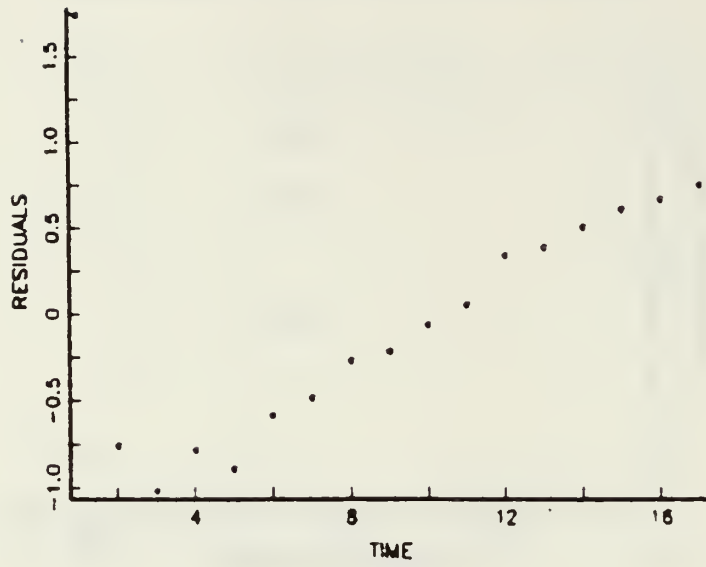
Figure 3. Ln Transformed Data Adjusted for Autocorrelation

modeling of autocorrelation into the data yielded similar results. Inspection of the probability plot, Figure 5, a symmetry plot of the residuals, the "rough cut" measure of normality (94 percent of the standardized residuals within the appropriate student's t value) and the Hotelling-Pabst statistic ( $T=286$ ,  $N=17$ ) supporting constant variance did not suggest major departures from the other distributional assumptions of the model. The results of the second linear regression are highlighted in Table V.

While considerable literature exists discussing the the need to retain the first observation for further



RESIDUALS VS TIME: 17 OBSERVATIONS



RESIDUALS VS FITTED VALUES: 17 OBSERVATIONS

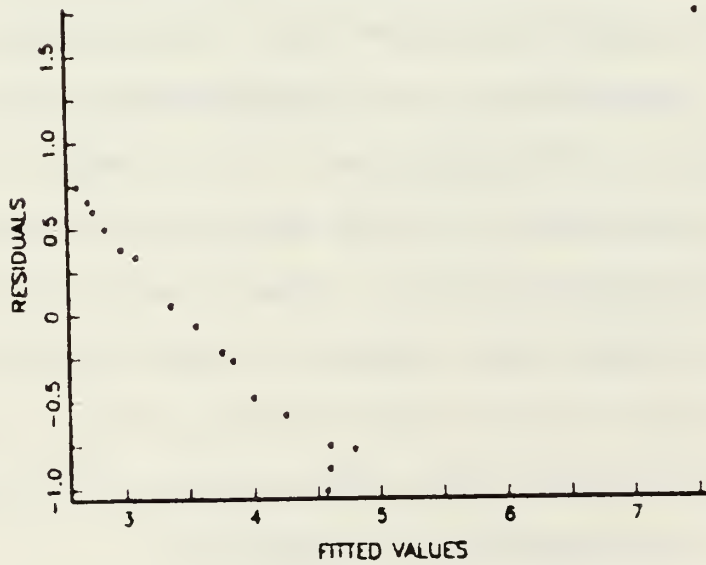


Figure 4. Residual Plots

TABLE V

## LINEAR REGRESSION 2 RESULTS

$\ln \hat{\beta}_0$  : 7.5234 (12.397,  $\alpha < .001$ )  
 $\hat{\beta}_1$  : -2.191 (-6.3097,  $\alpha < .001$ )  
D.W. : .78  
N : 17  
 $\hat{\rho}$  : .6054  
 $R^2$  : .73  
 $R^2$  adj. : .71

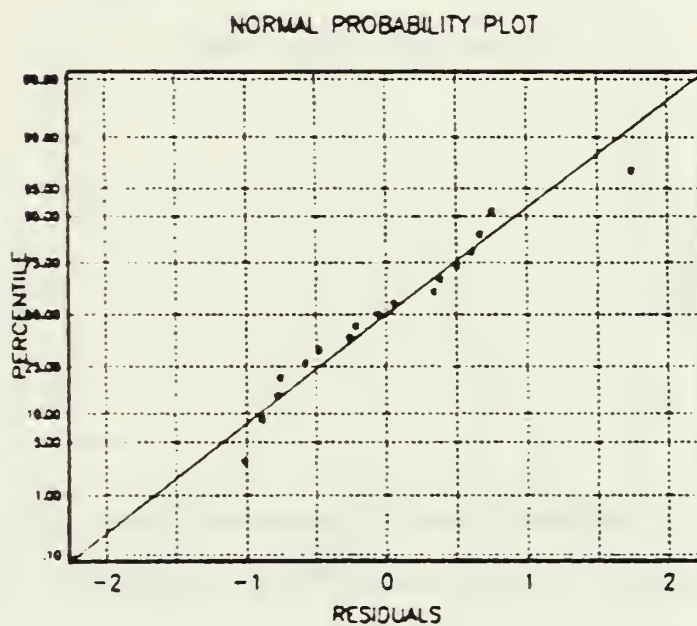


Figure 5. Normal Probability Plot of Residuals

regressions after autocorrelation has been modeled into the data, especially when sample size is small, the second regression resulted in an unexpected value for  $\beta_1$ . A third regression was performed after omitting the first observation to see what effects would be seen in parameter estimation and prediction results. The scatter plot of the residuals against time, Figure 6, appear to be more randomly scattered in a narrow horizontal band about zero. Furthermore, the probability plot and histogram, Figure 7, and the "rough cut" measure of normality (94 percent of the standardized residuals within the appropriate student's t value) support the distributional assumptions of the

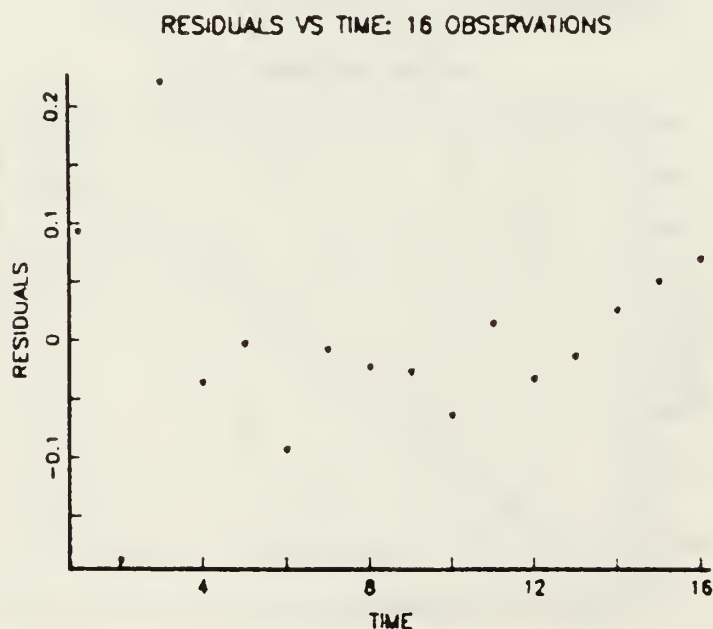


Figure 6. Residual Plot

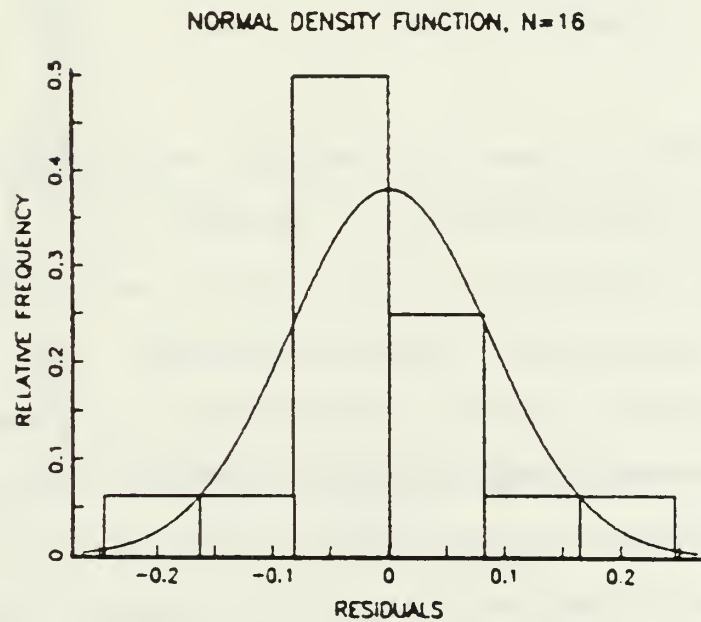
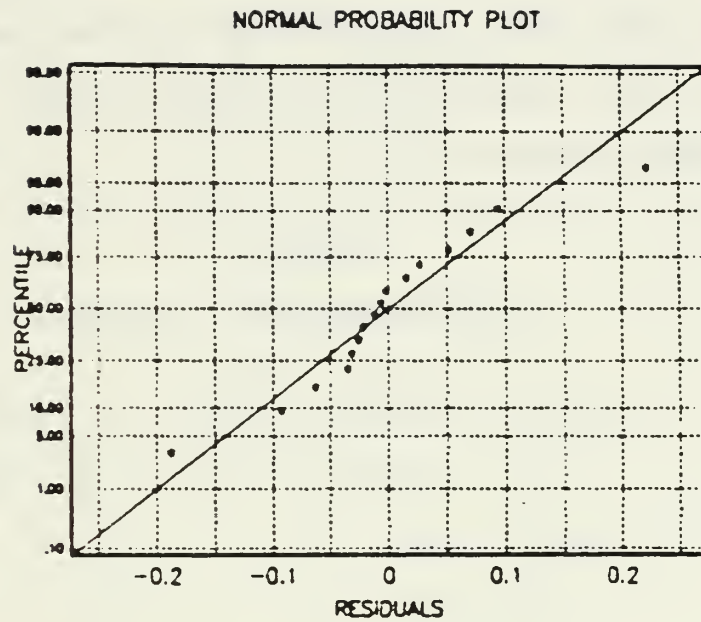


Figure 7. Normal Probability and Density Plots

model. The Durbin-Watson statistic and the test for homoskedasticity (Hotelling-Pabst statistic,  $T=572$ ,  $N=16$ ) suggested the other assumptions of the model were not violated. The results of the third linear regression are highlighted in Table VI.

TABLE VI  
LINEAR REGRESSION 3 RESULTS

|                     |   |        |                            |
|---------------------|---|--------|----------------------------|
| $\ln \hat{\beta}_0$ | : | 4.399  | (35.785, $\alpha < .001$ ) |
| $\hat{\beta}_1$     | : | -.4877 | (-7.146, $\alpha < .001$ ) |
| D.W.                | : | 2.9    |                            |
| N                   | : | 16     |                            |
| $\hat{\rho}$        | : | -.4730 |                            |
| $R^2$               | : | .78    |                            |
| $R^2$ adj.          | : | .77    |                            |

The nonlinear regressions were performed using 17 bivariate observations (2 through 18). The initial parameter estimates for  $\beta_0$  and  $\beta_1$  were taken from the results of the first linear regression. The other initial values required by the STATGRAPHICS nonlinear estimation panel used the system default values. The results of the first nonlinear regression are highlighted in Table VII. Inspection of the residuals plotted against time, Figure 8, and the Durbin-Watson statistic led to acceptance of the alternative hypothesis ( $H_1: \rho > 0$ ).

TABLE VII  
NONLINEAR REGRESSION 1 RESULTS

$\hat{\beta}_0$  : 491696.31  
 $\hat{\beta}_1$  : -.214  
D.W. : .86  
N : 17  
 $\hat{\rho}$  : .5629  
 $R^2$  : .96

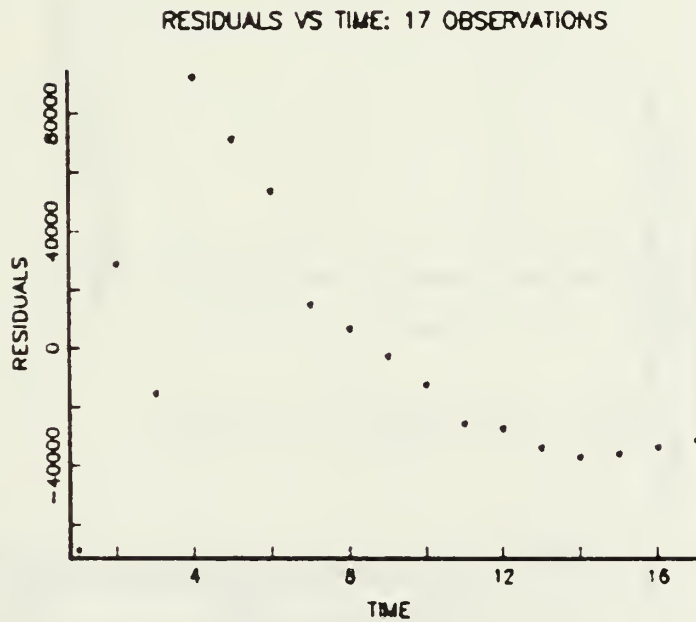


Figure 8. Residual Plot



The second nonlinear regression was performed on the same 17 bivariate observations after autocorrelation was modeled, Figure 9. The results of this regression are highlighted in Table VIII.

TABLE VIII  
NONLINEAR REGRESSION 2 RESULTS

$\hat{\beta}_0$  : 307094.63  
 $\hat{\beta}_1$  : -.382  
D.W. : 2.44  
N : 17  
 $\hat{\rho}$  : -.2371  
 $R_2$  : .94

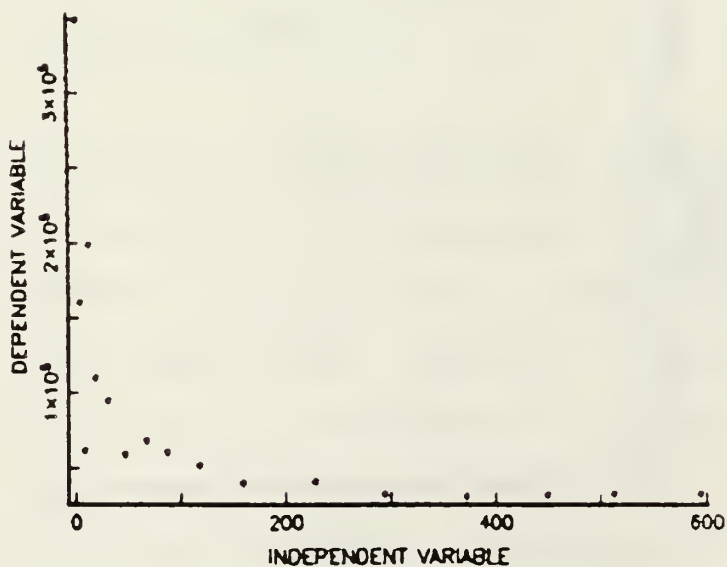


Figure 9. Ln Transformed Data, Autocorrelation Transformation, First Observation Omitted

Inspection of the residuals plotted against time, Figure 10, revealed the residuals to be patternless and lying in a narrow interval around zero. While the test for constant variance (Hotelling-Pabst statistic, T=878, N=17), the Durbin-Watson statistic and the "rough cut" measure of normality (94 percent of the standardized residuals within

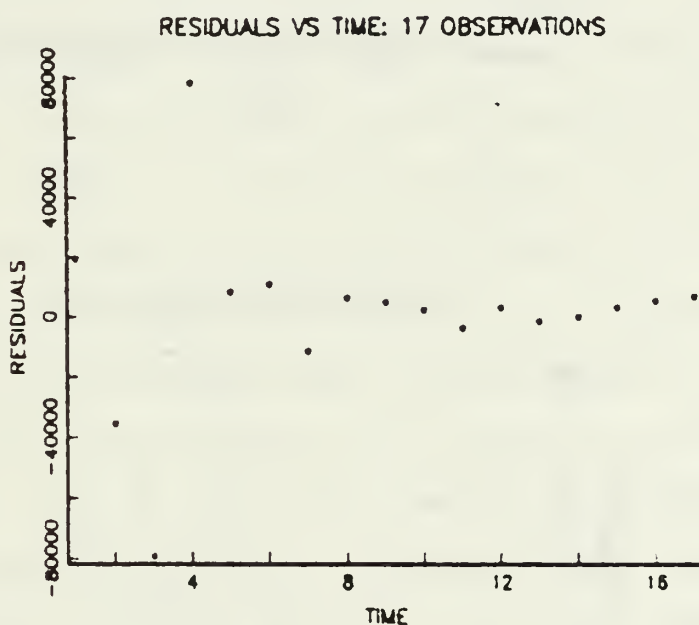


Figure 10. Residual Plot

the appropriate student's t value) support the assumptions of the model, the probability and density plots, Figure 11, suggest major departures from the assumption of normality of the error term. The implications of the residuals not

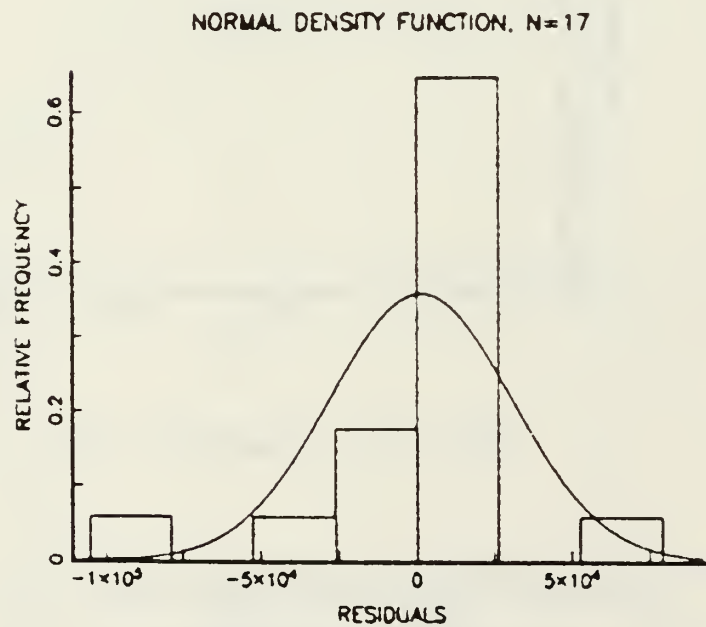
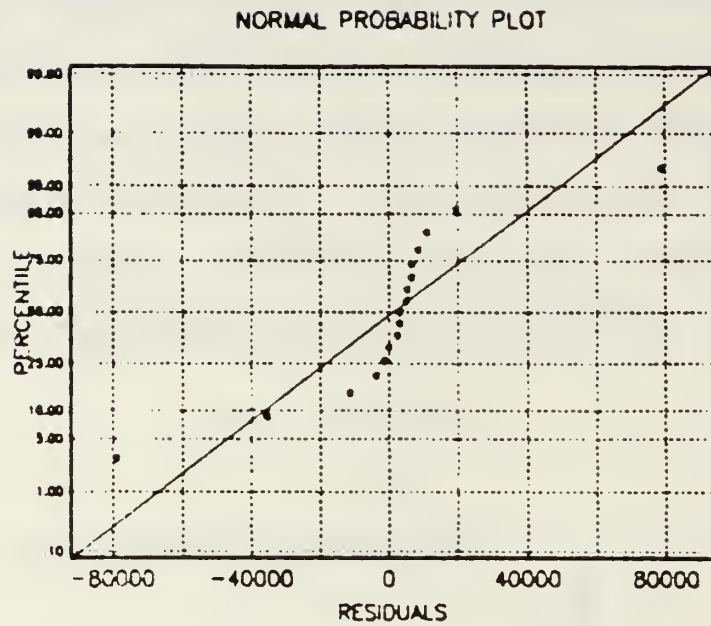


Figure 11. Normal Probability and Density Plots

reflecting the assumptions of the model will be discussed in the structure analysis portion of the results and in the conclusions.

## B. VALIDATION

The validation phase of this study consisted of a predictive analysis of the different model specifications and the regression techniques utilized. The initial investigation of the predictive ability of each case employed the prediction accuracy measures of MPE, MAPE, and the Pearson correlation coefficients adjusted for degrees of freedom. The results of these calculations are tabulated in Table IX. The predicted and fitted results of each model specification and regression method were transformed into the units of the original model specification, i.e., direct labor hours for the Xth unit for the unit learning curve, average cost per unit for the cumulative average learning curve, and the average cost in direct labor hours for the units produced in time period t for the Boger et. al. learning curve, prior to calculating the prediction accuracy measures. While the results for a model specification are comparable over the various regressions performed, the results are not directly comparable across model specifications.

The negative values for MPE reflected that the initial regression, linear or nonlinear, for each specification

TABLE IX

## PREDICTION ACCURACY MEASURES--ENTIRE HOLDOUT SAMPLE

|            | $n_d$      | $n_v$ | MPE | MAPE   | $R^2(\text{fitted})$ | $R^2(\text{validate})$ |        |
|------------|------------|-------|-----|--------|----------------------|------------------------|--------|
| F-102 Data | L Boger 1  | 17    | 5   | -72.56 | 72.56                | .291                   | -1.050 |
|            | L Boger 2  | 17    | 5   | 59.46  | 59.46                | -15.161                | -.052  |
|            | L Boger 3  | 16    | 5   | 61.58  | 61.58                | .047                   | .077   |
|            | NL Boger 1 | 17    | 5   | -89.42 | 89.42                | .587                   | -.284  |
|            | NL Boger 2 | 17    | 5   | 38.20  | 38.20                | .587                   | -.284  |
|            | L Cum 1    | 174   | 29  | -4.08  | 4.08                 | .972                   | .999   |
|            | NL Cum 1   | 174   | 29  | -6.40  | 6.40                 | .979                   | .999   |
|            | NL Cum 2   | 174   | 29  | 51.03  | 51.03                | .174                   | -1.530 |
|            | NL Cum 3   | 173   | 29  | 49.15  | 49.15                | .018                   | -1.536 |
|            | L Unit 1   | 173   | 29  | 1.64   | 5.78                 | .876                   | .914   |
|            | L Unit 2   | 173   | 29  | 57.73  | 57.73                | .562                   | .912   |
|            | L Unit 3   | 172   | 29  | 67.43  | 67.43                | .863                   | .893   |
| C-141 Data | NL Unit 1  | 173   | 29  | -3.29  | 5.81                 | .892                   | .915   |
|            | L Boger 1  | 17    | 6   | 1.50   | 3.81                 | .747                   | .996   |
|            | L Boger 2  | 17    | 6   | 67.30  | 67.30                | -15.95                 | .923   |
|            | L Boger 3  | 16    | 6   | 66.29  | 66.29                | .087                   | .915   |
|            | NL Boger 1 | 17    | 6   | -29.44 | 29.44                | .868                   | .996   |
|            | NL Boger 2 | 17    | 6   | 61.30  | 61.30                | .948                   | .960   |
|            | L Cum 1    | 9     | 3   | -2.97  | 2.97                 | .980                   | .999   |
|            | L Cum 2    | 9     | 3   | 46.32  | 46.32                | -13.96                 | .916   |
|            | L Cum 3    | 8     | 3   | 66.38  | 66.38                | -.481                  | .815   |
|            | NL Cum 1   | 9     | 3   | -8.51  | 8.51                 | .986                   | .999   |
|            | L Unit 1   | 9     | 3   | 12.92  | 12.92                | .976                   | .981   |
|            | NL Unit 1  | 9     | 3   | 4.43   | 6.12                 | .985                   | .981   |

where

- $n_d$  : number of developmental data points  
 $n_v$  : number of predicted data points  
L : linear regression model  
NL : nonlinear regression model  
Boger : Boger et. al learning curve specification  
Unit : Unit learning curve specification  
Cum : Cumulative average learning curve specification

overestimated the actual costs. On the other hand, after the transformation for autocorrelation was performed, the models usually underestimated the actual costs. The most striking feature of this table is the extremely large values of percent error after autocorrelation was modeled. This implied the predicted values severely underestimated the actual costs and could have been caused by predicting values too far outside the range of the developmental data. When the first observation was omitted following the adjustment for autocorrelation, the predictions were slightly more biased--but not by a large amount. Whereas the MPE for the Boger et. al. model, F-102 data, implied the model did not predict well at all; the MPE for the Boger et. al model, C-141 data, reflected excellent predictability. The Boger et. al model, F-102 data, MPE was not at all consistent with the MPE values for the unit and cumulative average learning curves using the F-102 data. Conversely, the Boger et. al. model, C-141 data, MPE was consistent with the results of the other specifications using the C-141 data. This observation could be due to unrealistic refinements to the data or the difference in sample size. After the transformation of the data for autocorrelation was made, the predicted values of the Boger et. al. model for both the C-141 and F-102 data were extremely high but consistent with the results of the other specifications. Another result that



was the MPE values for each of the nonlinear regressions (with no adjustment for autocorrelation) were larger than the respective linear regressions.

In most cases, MAPE was the absolute value of the respective MPE value. This implied that the models generally did not produce predictions that bracketed the actual values but rather predicted costs that were consistently either above or below the actual costs.

Prior to the data being adjusted for autocorrelation, the  $R^2$  (fitted) and  $R^2$  (validate) values were in the interval (.75, .99) except for the Boger et. al. model for the F-102 data. While the Boger et. al. linear and nonlinear models, C-141 data, had slightly larger differences of  $R^2$  square values than the other specifications (reflecting slightly more variability in prediction results) the Boger et. al. linear and nonlinear models, F-102 data, reflected extremely high variability of the fitted and predicted residuals relative to the variability of the dependent variable of the development data--which is not a desirable trait of a model. Negative values for  $R^2$  are indicative of cases where the sample variance of the residuals are higher than the sample variance of the developmental dependent variable. In all cases, when the autocorrelation transformation was incorporated, the  $R$  squared values decreased and the differences between  $R^2$  (fitted) and  $R^2$  (validate) grew larger.

The same prediction accuracy measures were calculated for predicted values not as far outside the developmental data range. These results are also tabulated in Table X. Whereas the MPE and MAPE values decreased slightly (except for the Boger et. al. model, F-102 data), the  $R^2$  values remained pretty much unchanged. The same trends described for the previous table apply to this table also. The implication of the results reflected in this table of calculations was the range of the predicted values outside the developmental range and had little effect on the initial prediction accuracy measures.

#### C. STRUCTURAL ANALYSIS

In most cases, the error process of the linear and nonlinear statistical models did not exhibit the desired normally distributed, random structure but, instead, exhibited a structure in which the error between adjacent observations were related to each other. As discussed above, the presence of autocorrelation in the residuals of a model results in biased estimates of the standard errors of the regression coefficients. Hence, the standard t-tests for significance of the difference of the estimates of the regression coefficients from zero, and the coefficients of determination may be erroneous.

In all cases where the Durbin-Watson test for autocorrelation resulted in accepting the alternative

TABLE X

## PREDICTION ACCURACY MEASURES--PORTION OF HOLDOUT SAMPLE

|            | Regression | $n_d$ | $n_v$ | MPE    | MAPE  | $R^2_f$ | $R^2_v$ |
|------------|------------|-------|-------|--------|-------|---------|---------|
| F-102 Data | L Boger 1  | 17    | 3     | -19.23 | 19.23 | .291    | .704    |
|            | L Boger 2  | 17    | 3     | 55.54  | 55.54 | -15.161 | -.998   |
|            | L Boger 3  | 16    | 3     | 56.61  | 56.61 | .047    | -.823   |
|            | NL Boger 1 | 17    | 3     | -30.33 | 30.33 | .333    | .725    |
|            | NL Boger 2 | 17    | 3     | 46.44  | 46.44 | .587    | -.301   |
|            | L Cum 2    | 174   | 10    | -3.81  | 3.81  | .972    | .999    |
|            | NL Cum 1   | 174   | 10    | -6.03  | 6.03  | .979    | .999    |
|            | NL Cum 2   | 174   | 10    | 24.90  | 24.90 | .173    | -1.55   |
|            | NL Cum 3   | 173   | 10    | 23.98  | 23.98 | .018    | -1.56   |
|            | L Unit 2   | 173   | 10    | -4.74  | 4.74  | .876    | .946    |
|            | L Unit 3   | 173   | 10    | 54.56  | 54.56 | .562    | .831    |
|            | L Unit 4   | 172   | 10    | 64.55  | 64.55 | .863    | .764    |
|            | NL Unit 1  | 173   | 10    | -9.85  | 9.85  | .892    | .946    |
|            | L Boger 1  | 17    | 4     | 1.96   | 5.22  | .747    | .989    |
| C-141 Data | L Boger 2  | 17    | 4     | 57.16  | 57.16 | -15.95  | .858    |
|            | L Boger 3  | 16    | 4     | 56.32  | 56.32 | .087    | .842    |
|            | NL Boger 1 | 17    | 4     | -28.47 | 28.47 | .868    | .989    |
|            | NL Boger 2 | 17    | 4     | 55.40  | 55.40 | .949    | .910    |
|            | L Cum 1    | 9     | 2     | -3.46  | 3.46  | .980    | .999    |
|            | L Cum 2    | 9     | 2     | -39.35 | 39.35 | -13.96  | .860    |
|            | L Cum 3    | 8     | 2     | 57.65  | 57.65 | -.481   | .682    |
|            | NL Cum 1   | 9     | 2     | -2.02  | 2.56  | .986    | .999    |
|            | L Unit 1   | 9     | 2     | 7.55   | 7.55  | .976    | .999    |
|            | NL Unit 1  | 9     | 2     | -1.26  | 1.26  | .985    | .999    |
|            |            |       |       |        |       |         |         |

hypothesis ( $H_1: \rho > 0$ ), this problem was addressed by modeling this phenomenon into the data and performing subsequent regressions. In every case, the  $R^2$  value of the regression decreased after modeling AR [1] and then increased after the first observation was omitted. Similarly, the t-statistics followed the same trend, and, in all cases, the estimated coefficients were statistically significant. The statistical significance of the estimated coefficients and the  $R^2$  values (listed in Appendices D, E, F, G, H, I) indicated that there is indeed a good amount of information contained in, and a good deal of the variation is explained by, the regression model.

After modeling the autocorrelation into the data and performing follow-on regressions, the nature of the residuals changed. The initial regression usually generated results that had a distinct cyclic pattern. The follow-on regressions reflected a linear pattern in two cases, but always a non-cyclic pattern--usually patternless.

In all cases after autocorrelation was modeled, the residuals also appeared to be and were statistically verified to be homoskedastic. Other distributional observations were made. In the small sample sizes ( $N=9$ , C-141, data unit and cumulative average models), the residuals of the follow-on regressions, both linear and nonlinear, met the "rough-cut" requirements for normality.

These normal assumptions were further reflected in the probability and symmetry plots and the estimated third and fourth moments. In the mid-sized samples (N=17, C-141 and F=102 Boger et. al. data), the residuals of the follow-on regressions reflected the normal assumptions through the "rough-cut" requirements, the probability and symmetry plots and the estimated third and fourth moments (except for the C-141 nonlinear regression for the Boger et. al. data). While the "rough-cut" requirements were met for the large sample sizes (N=173, F-102 unit and cumulative average data), the probability and symmetry plots and the estimated third and fourth moments suggested that major departures from the assumptions of normality existed. These inconsistent observations may be caused by either the differences in sample sizes, adjustments that were done to the data or poor models. It also appeared that the "rough-cut" measures of normality were not very discriminating.

#### D. COMPARISON OF FITTED MODELS

One of the secondary aspects of this research was to graphically compare the fitted models, both linear and nonlinear, against the observed developmental data in the units of the original models.

The fitted model results for the Boger et. al. model, linear and nonlinear regressions, C-141 data, are plotted in Figure 12. The observed independent variable of the



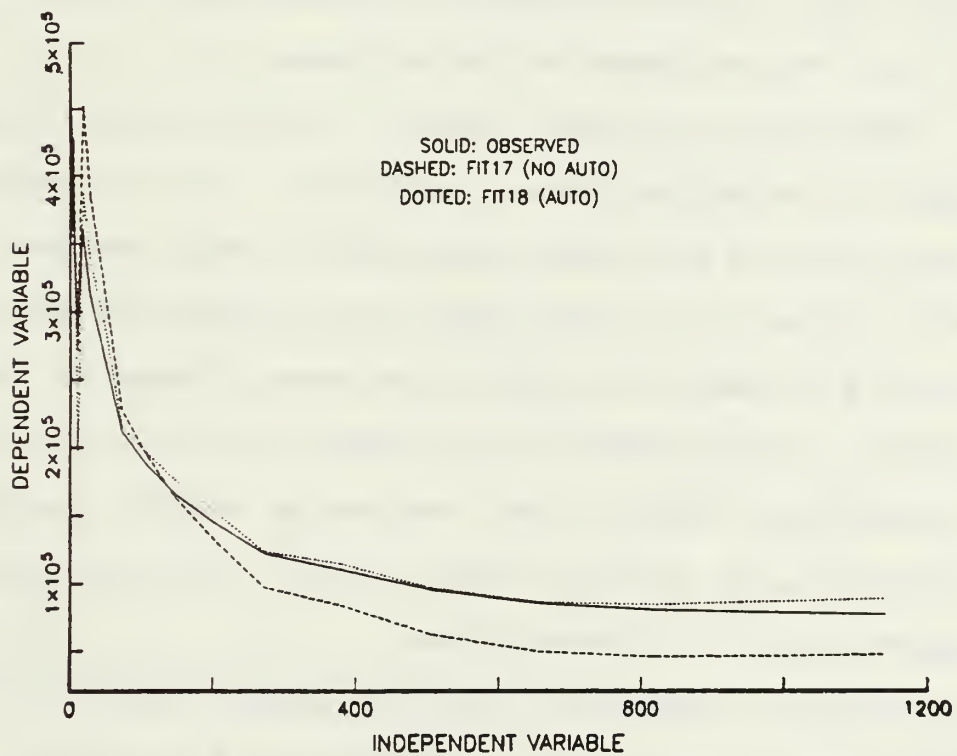
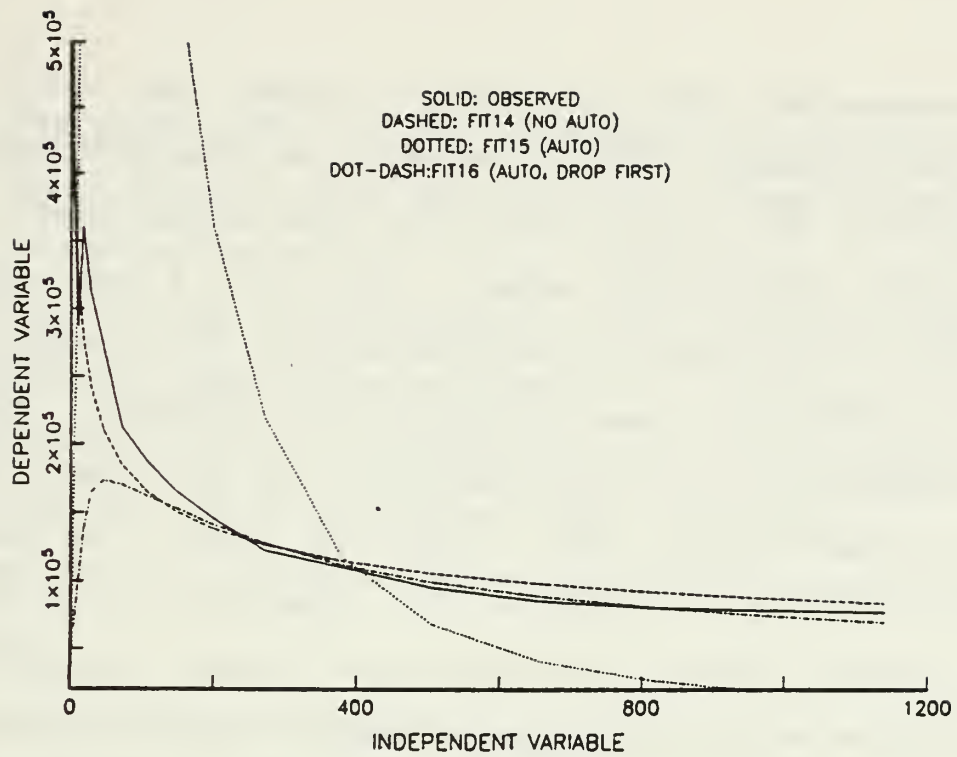


Figure 12. Fitted Model Results: Boger et. al. Model, C-141 Data



developmental data is plotted against the observed and fitted dependent variable values. As discussed above, the units for each fitted model have been transformed into the units of the original model. The initial linear regression with no autocorrelation modeled into the data, surprisingly, has a better fit than its nonlinear counterpart. After the transformation for autocorrelation was performed, however, the linear model had a poor fit while the nonlinear regression had an excellent fit. Whereas a third nonlinear regression was not performed, the linear regression with autocorrelation modeled and dropping the first observation had a poor initial fit but an excellent fit for the latter part of the developmental data range.

The remaining fitted models are listed in Appendix J. Generally speaking, the observations of each fitted model and regression technique were consistent across both sets of data. Prior to the adjustment for autocorrelation, both the linear and nonlinear regressions were comparable (except in the case of the Boger et. al. model, F-102 data). This was a surprising result since one would expect the nonlinear regression to have a much better fit than the linear regression for nonlinear data.

After the transformation for autocorrelation was made, the fitted linear models appeared to fit poorly. On the other hand, the fitted nonlinear models, while not as good

as the model prior to the adjustment for autocorrelation, appeared to have better fits than their linear counterparts.

Finally, after the initial observation was omitted following the transformation for autocorrelation, an interesting observation was noted. In all cases, the fitted model--both linear and nonlinear--was poor for the initial portion of the developmental data but appeared to be an excellent fit for the latter portion of the developmental data.

#### E. COMPARISON OF FITTED LOT COSTS

The cost for each lot derived from the fitted models for each of the regressions performed for both the C-141 and the F-102 data are plotted against the observed cost per lot in Appendix K. The fitted lot costs for the C-141 data are plotted for lots two through eight. Only these seven lots are plotted and used for comparison since omission of data points in some regressions and production data for a lot lying outside the developmental data range result in incomplete fitted lot costs. The fitted lot costs for the C-141 are plotted against the respective observed lot costs [see Ref. 5, p. 267] in each plot. The fitted lot costs for the F-102 data are plotted for lots four through nine for the same reasons cited above. The observed lot costs for the F-102 data are not the same for each plot since some outliers were initially identified and omitted (not always

the same points) prior to performing the regression. Inclusion of these outliers in the calculation of observed lot costs, in some cases, would bias the fitted lot costs down.

Visual inspection of the fitted lot costs plots, Appendix K, gives a good impression of the fit of each specification of the learning curve to the lot costs. Since each specification has been translated into fitted costs per lot, a basis exists for comparison across regression techniques and learning curve specifications. Figure 13 is an example of one plot of the fitted costs per lot for the Boger et. al. model, nonlinear regression, C-141 data.

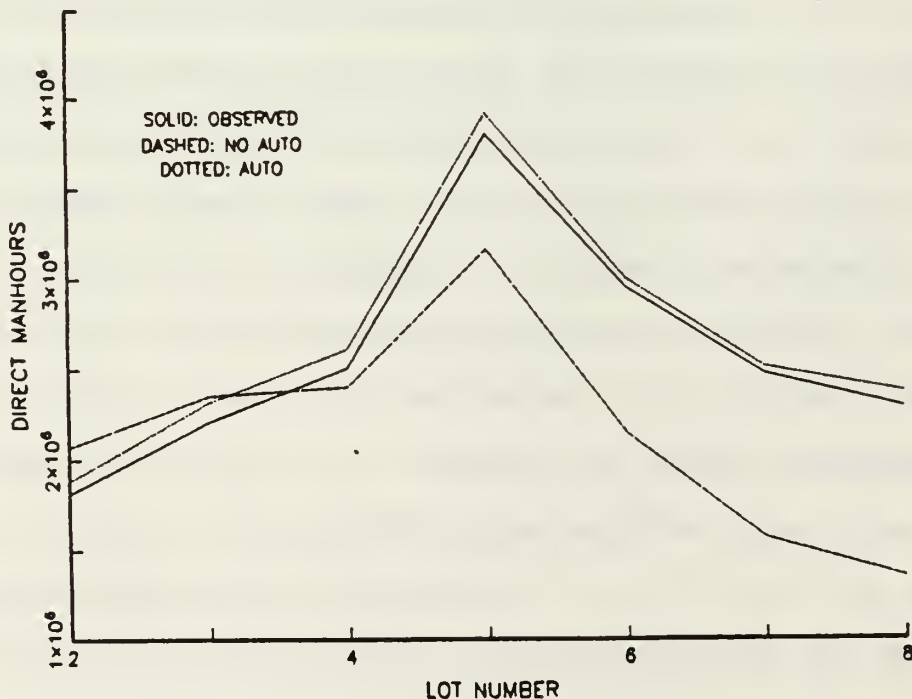


Figure 13. Fitted Lot Costs Results: Boger et. al. Model, Nonlinear Regression, C-141 Data

Whereas the initial regression does not appear to provide a good fit to the observed data, the fitted costs per lot after autocorrelation was modeled has a much better fit.

In general, the unit learning curve, both linear and nonlinear regression techniques, with and without transformations for autocorrelation, provided the best fitted lot costs for the F-102 production data. With respect to the cumulative average learning curve specification, the linear and nonlinear regressions without transformations for autocorrelation appear to have excellent lot cost fits--not as good as but comparable to the unit specification fits. The fitted lot costs for the cumulative average model, nonlinear regression with the transformation for autocorrelation, appear to have reasonable fits--but not as good as their unit specification counterparts. Whereas the linear regression for the Boger et. al. model appears to have a better fit than its nonlinear counterpart (except when autocorrelation is modeled) and a good fit overall, the fitted lot costs do not compare favorably with the cumulative average and the unit learning curve specifications. A nonparametric statistical test was then performed comparing the linear and nonlinear regression results, no autocorrelation modeled, of all three models. The purpose of this test was to statistically compare the fitted lot costs for each model. As discussed above, the Kruskal-Wallis test

was performed using the vectors of differences between the fitted lot costs and observed lot costs for the different models as the treatments. The null hypothesis (each model tends to yield identical residual lot costs) was rejected with the Kruskal-Wallis test statistic  $T = 19.07$ , 5 degrees of freedom,  $.001 \leq \alpha \leq .005$ . Multiple comparisons were then performed between models with  $\alpha = .05$ , 30 degrees of freedom. At this level, the Boger et. al. model, both linear and nonlinear regression results, tended to yield larger residual lot costs than both the unit and cumulative average models. The cumulative average and unit learning curve specifications tended to yield residual lot costs that were statistically equal.

With respect to the C-141 data, the unit learning curve specification, linear and nonlinear regressions, appear to have excellent fitted lot costs--seemingly better than the cumulative average and Boger et. al. specifications. The linear and nonlinear fitted lot costs of the cumulative average and Boger et. al. models, contrary to the F-102 data, compared favorably. A nonparametric statistical test was then performed comparing the linear and nonlinear regression results, no autocorrelation modeled, of all three models. The purpose of the test and data description are the same as above. The null hypothesis was rejected with the Kruskal-Wallis test statistic  $T = 13.22$ , 5 degrees of



freedom, with  $\alpha = .05$  and 36 degrees of freedom. At this level, the unit specification, linear regression, tended to yield smaller residual lot costs. All the other models tended to yield statistically equal residual lot costs.

Generally speaking, the linear models, with the transformation for autocorrelation performed, resulted in very poor lot cost fits for both sets of data. On the other hand, the nonlinear regressions with autocorrelation modeled resulted in reasonable fits. Similarly, when the first observation was omitted after modeling autocorrelation, the fitted lot costs were reasonable for both the linear and nonlinear regression techniques.



## VI. CONCLUSIONS

The primary purpose of this research was to empirically investigate the validity of a reformulation of the cumulative average learning curve derived and discussed by Boger, Jones and Sontheimer in "Budgets, Contracts, Incentives and Costs: A Stylized Nexus" [Ref. 14:p. 23]. In the process of conducting this investigation, the impacts of linear versus nonlinear regression methods and modeling autocorrelation were also addressed.

The linear and nonlinear Boger et. al. models for both sets of data, before autocorrelation was modeled, while not as good as the fitted cumulative average and unit learning curve models, did not suggest gross inadequacies. Similarly, the fitted cost per lot for the Boger et. al. model, while statistically different from the cumulative average and unit specifications for the F-102 data, was not statistically different from the cumulative average model for the C-141 data. Again, the plots of the fitted costs per lot did not suggest gross inadequacies of the Boger et. al. model.

Surprisingly, it was also noted that the nonlinear regressions did not consistently provide much better fitted models and fitted lot costs. Also, in agreement with other

literature and research, the unit learning curve specification generally provided better fitted models and fitted lot costs than both other models.

The predictive ability of the Boger et. al. model for the C-141 data was consistent with the cumulative average and unit specification. This was not true for the F-102 data and is partly blamed on the noise in the data in the case of the Boger et. al. model.

Whenever autocorrelation was modeled into the data, poorly fitted lot costs emerged in the linear regression cases. On the other hand, when autocorrelation was modeled during the nonlinear regressions, the results were not substantially degraded. The predictive ability of all models was adversely affected when the autocorrelation was modeled. Areas for further research would include other methods of autocorrelation modeling and the effects that other estimates of  $\rho$  might have.

While the structure of the residuals did not always reflect the assumptions of the model being analyzed, which might lead one to consider rejecting the model, Pesaran cautions:

There is not theoretical justification for expecting a correctly specified model to possess all the characteristics of the classical regression models. The assumptions underlying the classical regression models are made, not because they are optimal from the point of view of economic theory, but because they are extremely convenient for estimation and hypothesis testing purposes. [Ref. 24:p. 154]

While observing some contradictory results between the two sets of aircraft production data, this researcher feels that the results generally suggest the Boger et. al. learning curve specification is an adequate model. This conclusion is tempered by several observations. It is felt that the C-141 and F-102 data used was a severe limitation to the scope of this study. While the sample size of the F-102 data was generally large enough for the analysis, the adjustments made to the data to meet the form required by the Boger et. al. model (discussed in detail by Womer and Gullede [Ref. 12:p. 81]) are rough approximations and have introduced considerable noise into the data. On the other hand, whereas the data for the C-141 analysis appeared to be very smooth, the small sample size was a limitation. This researcher feels that a more conclusive analysis could be performed with considerably more effort going into the data gathering stage with dialogue between the analyst and the data source. Finally, the adjustments made to the data for the Boger et. al. model in this study used equivalent units produced per time period based on approximate production rates to generate the independent and dependent variables. Other proxy variables might also be worth investigating.

# APPENDIX A

## ADJUSTED CUMULATIVE AVERAGE LEARNING CURVE DATA

### C-141 DATA

| INDEP | DEPEN       |
|-------|-------------|
| 5     | 423225.4    |
| 11    | 357139.3909 |
| 21    | 292402.7619 |
| 36    | 240278.9694 |
| 66    | 188599.7909 |
| 94    | 163846.0798 |
| 122   | 146553.9418 |
| 150   | 134506.1453 |
| 184   | 124547.1429 |
| 217   | 117384.8157 |
| 250   | 111631.5908 |
| 284   | 108303.619  |

### F-102 DATA

| INDEP | DEPEN       |
|-------|-------------|
| 1     | 198785.6875 |
| 2     | 191219.6804 |
| 3     | 185213.8301 |
| 4     | 173896.6535 |
| 5     | 167952.226  |
| 6     | 161332.6875 |
| 7     | 161401.8577 |
| 8     | 157705.6925 |
| 9     | 156184.7955 |
| 10    | 151562.5646 |
| 11    | 149342.8082 |
| 12    | 145756.1199 |
| 13    | 145005.0999 |
| 14    | 141416.6299 |
| 15    | 138694.999  |
| 16    | 135473.3503 |
| 17    | 134044.4885 |
| 18    | 132746.108  |
| 19    | 131026.6435 |
| 20    | 128931.8526 |
| 21    | 127743.2604 |
| 22    | 126916.129  |
| 23    | 125731.7915 |
| 24    | 126346.4326 |
| 25    | 125079.5524 |
| 26    | 124055.6411 |
| 27    | 122825.3994 |
| 28    | 121565.7707 |
| 29    | 120297.7286 |
| 30    | 119340.3851 |
| 31    | 118120.5626 |
| 32    | 117616.1813 |
| 33    | 116865.7062 |
| 34    | 115882.5315 |
| 35    | 115336.4696 |
| 36    | 114716.1899 |
| 37    | 113787.6912 |
| 38    | 113241.2048 |
| 39    | 112406.6125 |
| 40    | 111766.7878 |
| 41    | 111450.0828 |
| 42    | 110949.7169 |
| 43    | 110506.8852 |
| 44    | 109814.2177 |

|     |             |
|-----|-------------|
| 45  | 109274.077  |
| 46  | 108645.3467 |
| 47  | 107997.0418 |
| 48  | 107322.9439 |
| 49  | 107738.7195 |
| 50  | 107099.1025 |
| 51  | 106447.8066 |
| 52  | 105813.0005 |
| 53  | 105207.2654 |
| 54  | 104588.9196 |
| 55  | 103950.844  |
| 56  | 103558.5122 |
| 57  | 102967.6076 |
| 58  | 102511.1905 |
| 59  | 102119.26   |
| 60  | 101624.3641 |
| 61  | 101068.8712 |
| 62  | 100587.5227 |
| 63  | 100092.0185 |
| 64  | 99607.15497 |
| 65  | 99165.3994  |
| 66  | 98724.14488 |
| 67  | 98249.21286 |
| 68  | 97811.40959 |
| 69  | 97359.22169 |
| 70  | 96920.15013 |
| 71  | 96502.682   |
| 72  | 96487.96392 |
| 73  | 96062.59266 |
| 74  | 95646.50853 |
| 75  | 95231.91679 |
| 76  | 94936.76384 |
| 77  | 94523.38365 |
| 78  | 94115.95844 |
| 79  | 93684.01389 |
| 80  | 93288.39254 |
| 81  | 92867.76904 |
| 82  | 92492.23391 |
| 83  | 92125.37459 |
| 84  | 91759.38259 |
| 85  | 91397.79078 |
| 86  | 91065.10999 |
| 87  | 90709.44521 |
| 88  | 90592.83559 |
| 89  | 90250.72112 |
| 90  | 89921.49648 |
| 91  | 89609.36075 |
| 92  | 89300.22251 |
| 93  | 89275.92278 |
| 94  | 88962.94892 |
| 95  | 88910.61425 |
| 96  | 88606.39735 |
| 97  | 88336.35269 |
| 98  | 88055.24135 |
| 99  | 87726.11904 |
| 100 | 87417.2353  |
| 101 | 87114.22101 |
| 102 | 86810.91376 |
| 103 | 86504.70718 |
| 104 | 86208.22007 |
| 105 | 85924.00209 |
| 106 | 85643.41758 |
| 107 | 85361.04086 |
| 108 | 85082.73006 |
| 109 | 84818.42297 |
| 110 | 84558.93712 |
| 111 | 84306.83222 |
| 112 | 84056.04074 |
| 113 | 83793.9249  |
| 114 | 83521.40167 |
| 115 | 83276.40252 |
| 116 | 83039.14368 |
| 117 | 82780.02993 |
| 118 | 82559.27581 |
| 119 | 82464.90296 |



|     |               |
|-----|---------------|
| 120 | 822229.787779 |
| 121 | 81982.89915   |
| 122 | 81755.64561   |
| 123 | 81543.97334   |
| 124 | 81323.88326   |
| 125 | 81120.26343   |
| 126 | 80903.43738   |
| 127 | 80692.32284   |
| 128 | 80468.89023   |
| 129 | 80249.76885   |
| 130 | 80233.93735   |
| 131 | 80213.40972   |
| 132 | 80221.90155   |
| 133 | 80015.78865   |
| 134 | 79808.75137   |
| 135 | 79618.02026   |
| 136 | 79400.12205   |
| 137 | 79222.23675   |
| 138 | 79019.00689   |
| 139 | 78817.75404   |
| 140 | 78631.05443   |
| 141 | 78454.55837   |
| 142 | 78265.98632   |
| 143 | 78094.23485   |
| 144 | 77921.46865   |
| 145 | 77758.40858   |
| 146 | 77591.65301   |
| 147 | 77425.91937   |
| 148 | 77257.31471   |
| 149 | 77075.78451   |
| 150 | 76907.31623   |
| 151 | 76742.91998   |
| 152 | 76561.9087    |
| 153 | 76380.34463   |
| 154 | 76217.64448   |
| 155 | 76045.98488   |
| 156 | 75886.34457   |
| 157 | 76006.2876    |
| 158 | 75838.75124   |
| 159 | 75668.5868    |
| 160 | 75500.17295   |
| 161 | 75321.13076   |
| 162 | 75173.77906   |
| 163 | 75008.63919   |
| 164 | 74866.05503   |
| 165 | 74729.17832   |
| 166 | 74562.26775   |
| 167 | 74439.98741   |
| 168 | 74300.68973   |
| 169 | 74163.91122   |
| 170 | 73998.86733   |
| 171 | 73855.24838   |
| 172 | 73721.23904   |
| 173 | 73573.65304   |
| 174 | 73414.80647   |
| 175 | 73276.25392   |
| 176 | 73130.78816   |
| 177 | 72981.46278   |
| 178 | 72852.35505   |
| 179 | 72720.96864   |
| 180 | 72573.00981   |
| 181 | 72442.27472   |
| 182 | 72306.35706   |
| 183 | 72162.12077   |
| 184 | 72037.29362   |
| 185 | 71911.28561   |
| 186 | 71773.09122   |
| 187 | 71642.90453   |
| 188 | 71544.09864   |
| 189 | 71442.18396   |
| 190 | 71347.40422   |
| 191 | 71252.98716   |
| 192 | 71145.75328   |
| 193 | 71072.64149   |
| 194 | 70978.74376   |



|     |             |
|-----|-------------|
| 195 | 70868.34239 |
| 196 | 70763.42668 |
| 197 | 70672.20644 |
| 198 | 70576.38583 |
| 199 | 70485.34964 |
| 200 | 70388.27422 |
| 201 | 70308.3394  |
| 202 | 70221.55658 |
| 203 | 70123.806   |
| 204 | 70035.8303  |

# APPENDIX B

## ADJUSTED UNIT LEARNING CURVE DATA

### C-141 DATA

| INDEP       | DEPEN       |
|-------------|-------------|
| 2.3803364   | 423225.4    |
| 8.24566607  | 302067.7167 |
| 16.139209   | 221192.47   |
| 28.54110847 | 167305.66   |
| 50.46156288 | 126584.7767 |
| 79.92929653 | 105498.0464 |
| 108.0780045 | 88501.76429 |
| 136.165075  | 82012.175   |
| 167.0975761 | 80610.36765 |
| 200.68436   | 77449.41515 |
| 233.7289909 | 73799.77879 |
| 267.2484024 | 83833.23824 |

### F-102 DATA

| INDEP | DEPEN       |
|-------|-------------|
| 1     | 198785.6875 |
| 2     | 183653.6733 |
| 3     | 173202.1294 |
| 4     | 139945.124  |
| 5     | 144174.516  |
| 6     | 128234.9949 |
| 7     | 161816.879  |
| 8     | 131832.5362 |
| 9     | 144017.6192 |
| 10    | 109962.4866 |
| 11    | 127145.2443 |
| 12    | 106302.5488 |
| 13    | 135992.8595 |
| 14    | 94766.5199  |
| 15    | 100592.1663 |
| 16    | 87148.6196  |
| 17    | 111182.7003 |
| 18    | 110673.6384 |
| 19    | 100076.2828 |
| 20    | 89130.8264  |
| 21    | 103971.4164 |
| 22    | 109546.369  |
| 23    | 99676.3665  |
| 24    | 140483.1777 |
| 25    | 94674.4283  |
| 26    | 98457.8582  |
| 27    | 90839.1148  |
| 28    | 87555.7956  |
| 29    | 84792.5512  |
| 30    | 91577.4219  |
| 31    | 81525.8892  |
| 32    | 101980.3619 |
| 33    | 92850.503   |
| 34    | 83437.7656  |
| 35    | 96770.3649  |
| 36    | 93006.4016  |
| 37    | 80361.736   |
| 38    | 93021.208   |
| 39    | 80692.1038  |
| 40    | 86813.6248  |
| 41    | 98781.8842  |
| 42    | 90434.715   |
| 43    | 91907.9518  |

|     |            |
|-----|------------|
| 44  | 80029.5174 |
| 45  | 85507.8854 |
| 46  | 80352.482  |
| 47  | 78175.0158 |
| 48  | 75640.3452 |
| 49  | 127695.946 |
| 50  | 75757.871  |
| 51  | 73883.0106 |
| 52  | 73437.8932 |
| 53  | 73709.0354 |
| 54  | 71816.5924 |
| 55  | 69494.7638 |
| 56  | 81980.2606 |
| 57  | 69876.954  |
| 58  | 76495.4148 |
| 59  | 79387.2898 |
| 60  | 72425.5056 |
| 61  | 67739.3    |
| 62  | 71225.2618 |
| 63  | 69370.7602 |
| 64  | 69060.7512 |
| 65  | 70893.0432 |
| 66  | 70042.6006 |
| 67  | 66903.7    |
| 68  | 68478.59   |
| 69  | 66610.4445 |
| 70  | 66624.2125 |
| 71  | 67279.9135 |
| 72  | 95442.9798 |
| 73  | 65435.862  |
| 74  | 65272.367  |
| 75  | 64552.1285 |
| 76  | 72800.2926 |
| 77  | 63106.4885 |
| 78  | 62744.218  |
| 79  | 59992.339  |
| 80  | 62034.3055 |
| 81  | 59217.889  |
| 82  | 62073.8885 |
| 83  | 62042.9105 |
| 84  | 61382.0465 |
| 85  | 61024.0785 |
| 86  | 62787.243  |
| 87  | 60122.2745 |
| 88  | 80447.7982 |
| 89  | 60144.6475 |
| 90  | 60620.504  |
| 91  | 61517.145  |
| 92  | 61168.6425 |
| 93  | 87040.3478 |
| 94  | 59856.38   |
| 95  | 83991.1548 |
| 96  | 59705.7925 |
| 97  | 62412.065  |
| 98  | 60787.441  |
| 99  | 55472.1325 |
| 100 | 56837.746  |
| 101 | 56812.7915 |
| 102 | 56176.882  |
| 103 | 55271.636  |
| 104 | 55670.0475 |
| 105 | 56365.3315 |
| 106 | 56182.045  |
| 107 | 55429.1075 |
| 108 | 55303.4745 |
| 109 | 56273.258  |
| 110 | 56274.979  |
| 111 | 56575.2935 |
| 112 | 56218.186  |
| 113 | 54436.951  |
| 114 | 52726.277  |
| 115 | 55346.4995 |
| 116 | 55754.3765 |
| 117 | 52722.835  |
| 118 | 56731.044  |

|     |             |
|-----|-------------|
| 119 | 713228.9066 |
| 120 | 54251.083   |
| 121 | 52356.262   |
| 122 | 54257.967   |
| 123 | 55719.9565  |
| 124 | 54252.804   |
| 125 | 55871.4045  |
| 126 | 53800.181   |
| 127 | 54091.8905  |
| 128 | 52092.949   |
| 129 | 52202.2325  |
| 130 | 78191.673   |
| 131 | 77544.8184  |
| 132 | 81334.3314  |
| 133 | 52808.885   |
| 134 | 52272.7935  |
| 135 | 54060.052   |
| 136 | 49983.8635  |
| 137 | 55029.8355  |
| 138 | 51176.5165  |
| 139 | 51044.86    |
| 140 | 52679.81    |
| 141 | 53745.109   |
| 142 | 51677.3275  |
| 143 | 53705.526   |
| 144 | 53215.9015  |
| 145 | 54277.7585  |
| 146 | 53412.0955  |
| 147 | 53228.809   |
| 148 | 52472.4295  |
| 149 | 50209.3145  |
| 150 | 51805.542   |
| 151 | 52083.4835  |
| 152 | 49229.205   |
| 153 | 48782.6055  |
| 154 | 51324.5225  |
| 155 | 49610.4065  |
| 156 | 51142.0965  |
| 157 | 94717.4     |
| 158 | 49535.543   |
| 159 | 48782.6055  |
| 160 | 48722.3705  |
| 161 | 46674.3805  |
| 162 | 51450.1555  |
| 163 | 48255.9795  |
| 164 | 51624.837   |
| 165 | 52281.3985  |
| 166 | 47022.0225  |
| 167 | 54141.4524  |
| 168 | 51037.976   |
| 169 | 51185.1215  |
| 170 | 46106.4505  |
| 171 | 49440.0275  |
| 172 | 50805.641   |
| 173 | 48188.8605  |
| 174 | 45934.3505  |
| 175 | 49168.1095  |
| 176 | 47674.2815  |
| 177 | 46700.1955  |
| 178 | 50000.2874  |
| 179 | 49334.186   |
| 180 | 46088.38    |
| 181 | 48909.9595  |
| 182 | 47705.2595  |
| 183 | 45911.117   |
| 184 | 49193.9245  |
| 185 | 48725.8125  |
| 186 | 46207.129   |
| 187 | 47428.1785  |
| 188 | 53067.399   |
| 189 | 52282.2223  |
| 190 | 53434.0345  |
| 191 | 53313.7451  |
| 192 | 50664.0827  |
| 193 | 57035.1782  |

|     |            |
|-----|------------|
| 194 | 52856.4806 |
| 195 | 49450.478  |
| 196 | 50304.8623 |
| 197 | 52793.0403 |
| 198 | 51699.725  |
| 199 | 52460.1847 |
| 200 | 51070.2654 |
| 201 | 54321.3748 |
| 202 | 52778.2101 |
| 203 | 50378.1894 |
| 204 | 52176.7631 |

# APPENDIX C

## ADJUSTED BOGER ET AL LEARNING CURVE DATA

### F-102 DATA

| INDEP   | DEPEN   |
|---------|---------|
| 0       | 33688.1 |
| 1       | 47146.3 |
| 2.38077 | 56338.9 |
| 4.56154 | 45254   |
| 7.60362 | 52550.1 |
| 11.5382 | 62859.9 |
| 15.9344 | 52671   |
| 21.2575 | 58556.1 |
| 23.7728 | 50268.1 |
| 27.3882 | 54428.4 |
| 31.5228 | 38313.2 |
| 37.7959 | 40203.7 |
| 42.9844 | 40016.9 |
| 49.0382 | 37349.3 |
| 57.1344 | 34468.3 |
| 66.1921 | 33106.3 |
| 74.8959 | 26587.1 |
| 85.1844 | 26044.2 |
| 108.261 | 26926   |
| 131.723 | 30284.2 |
| 144.223 | 27512.2 |
| 152.915 | 17550   |
| 157.761 | 10236.9 |

### C-141 DATA

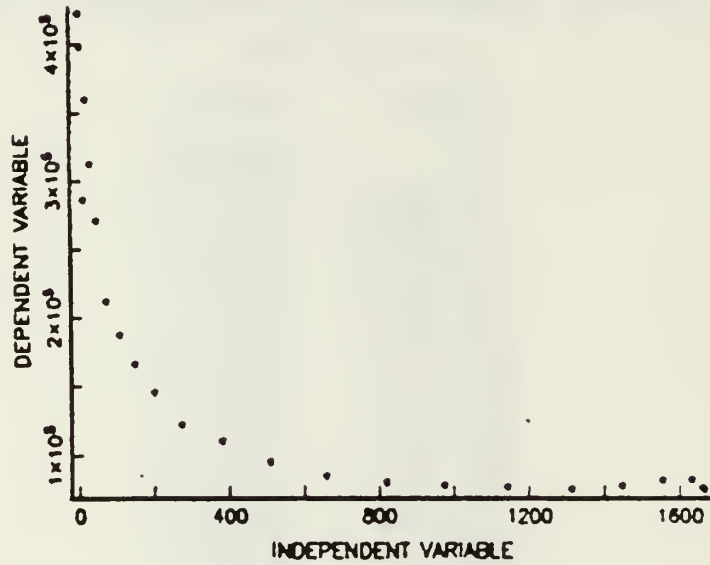
| INDEP   | DEPEN   |
|---------|---------|
| 0       | 423226  |
| 1       | 423234  |
| 3.79604 | 398778  |
| 9.77984 | 286633  |
| 17.1062 | 360400  |
| 27.5982 | 313197  |
| 45.3044 | 271292  |
| 71.6374 | 212284  |
| 106.329 | 187897  |
| 145.647 | 166776  |
| 199.061 | 146092  |
| 270.786 | 122850  |
| 379.058 | 110953  |
| 505.918 | 96014.4 |
| 655.962 | 86009.6 |
| 817.355 | 81303.6 |
| 971.643 | 79377.5 |
| 1140.14 | 78052.2 |
| 1308.93 | 76578.5 |
| 1444.26 | 79039.1 |
| 1551.81 | 82908.3 |
| 1629.37 | 83169.5 |
| 1658.49 | 77371.7 |
| 1663.95 | 75771   |



APPENDIX D

BOGER ET AL MODEL: C-141 DATA ANALYSIS RESULTS

RAW DATA: 18 OBSERVATIONS



LN TRANSFORMED DATA: 17 OBSERVATIONS

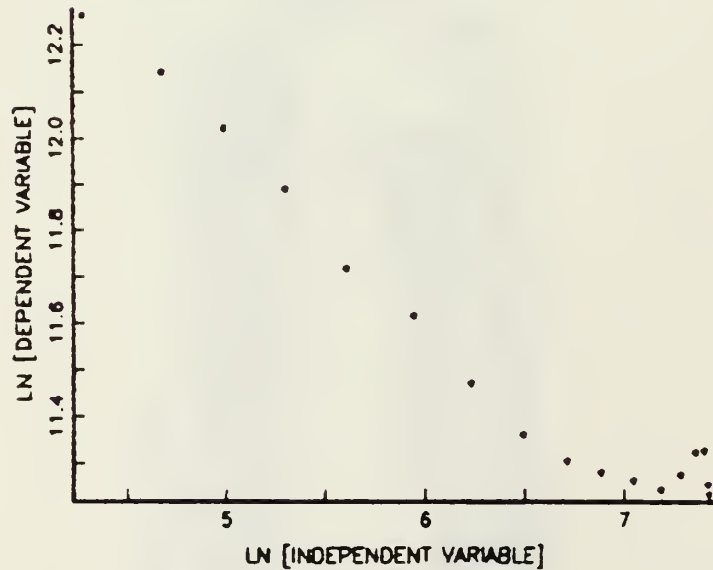


Figure D-1. Boger et al Specification: C-141 Data

TABLE D-1

## LINEAR REGRESSION RESULTS

|                                       | $\ln \beta_0$ | $(t\text{-stat})$ | $\beta_1$ | $(t\text{-stat})$ | D.W. | $\hat{\rho}$ | N  | $R^2$ | $R^2 \text{ adj}$ |
|---------------------------------------|---------------|-------------------|-----------|-------------------|------|--------------|----|-------|-------------------|
| ln transformed data                   | 13.332        | (123.21)          | -.2821    | (-13.00)          | .59  | .6987        | 17 | .92   | .91               |
| AR [1] modeled                        | 7.523         | (12.40)           | -2.191    | (-6.31)           | .78  | .6054        | 17 | .73   | .71               |
| AR [1] modeled,<br>first obs. omitted | 4.399         | (35.78)           | -.4877    | (-7.15)           | 2.9  | -.47         | 16 | .78   | .77               |

TABLE D-2

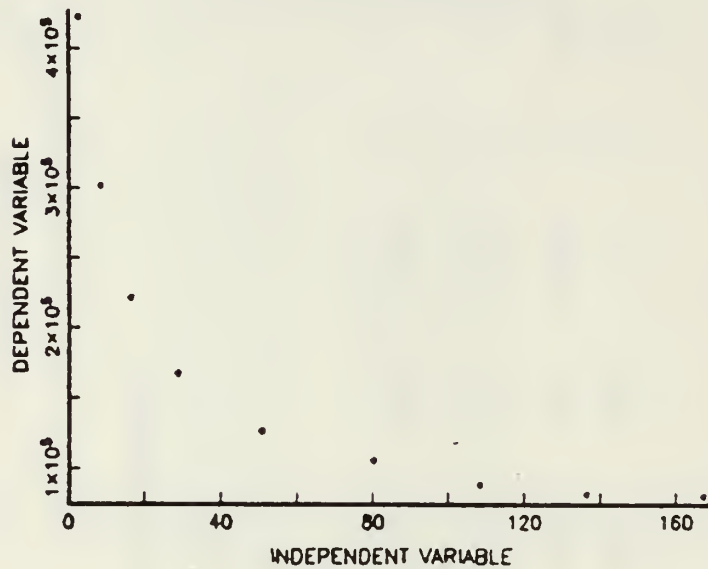
## NON-LINEAR REGRESSION RESULTS

|                | $\beta_0$ | $\beta_1$ | D.W. | $\hat{\rho}$ | N  | $R^2$ |
|----------------|-----------|-----------|------|--------------|----|-------|
| Paw data       | 491696.31 | -.214     | .86  | .5629        | 17 | .96   |
| AR [1] modeled | 307094.63 | -.382     | 2.44 | -.2371       | 17 | .94   |

# APPENDIX E

## UNIT LEARNING CURVE: C-141 DATA ANALYSIS RESULTS

RAW DATA: 9 OBSERVATIONS



LN TRANSFORMED DATA: 9 OBSERVATIONS

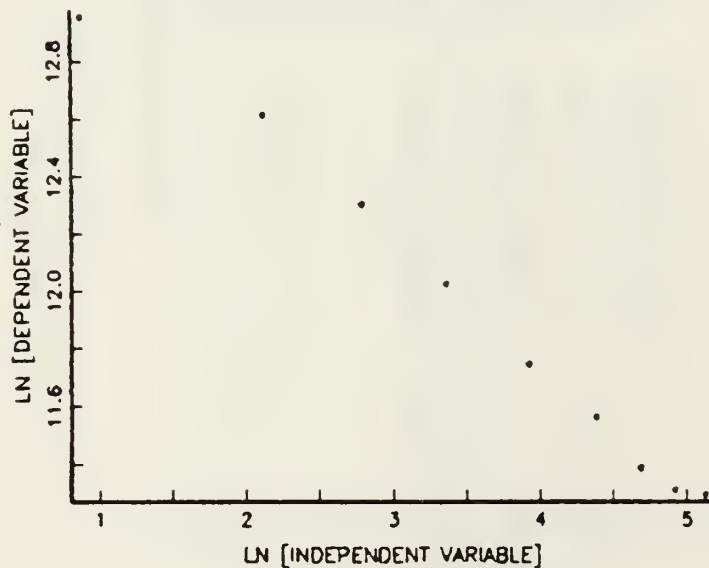


Figure E-1. Unit Learning Curve: C-141 Data

TABLE E-1  
LINEAR REGRESSION RESULTS

ln Transformed Data  
ln  $\beta_0$  : 13.414 (228.12)  
 $\beta_1$  : -.4201 (-27.27)  
D.W. : 1.69  
N : 9  
 $\hat{\rho}$  : .1121  
 $R^2$  : .92  
 $R^2$  adj : .91

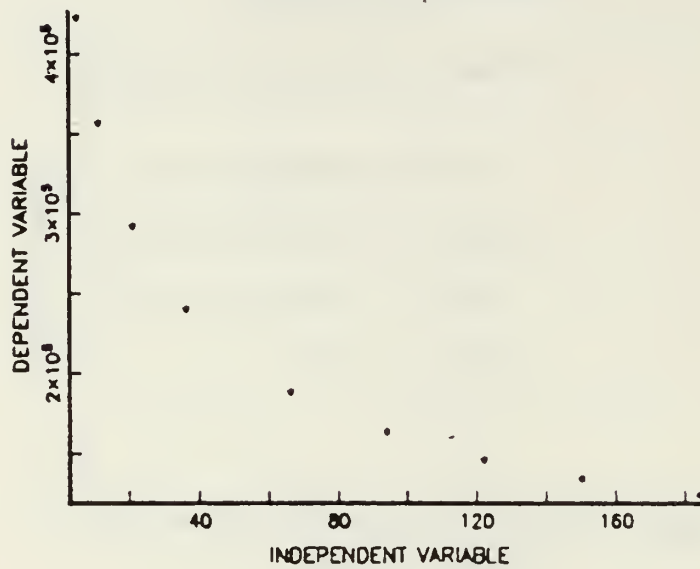
TABLE E-2  
NONLINEAR REGRESSION RESULTS

Raw Data  
 $\beta_0$  : 613600.37  
 $\beta_1$  : -.387  
D.W. : 1.64  
N : 9  
 $\hat{\rho}$  : .1374  
 $R^2$  : .996

# APPENDIX F

## CUMULATIVE AVERAGE LEARNING CURVE: C-141 DATA ANALYSIS RESULTS

RAW DATA: 9 OBSERVATIONS



LN TRANSFORMED DATA: 9 OBSERVATIONS

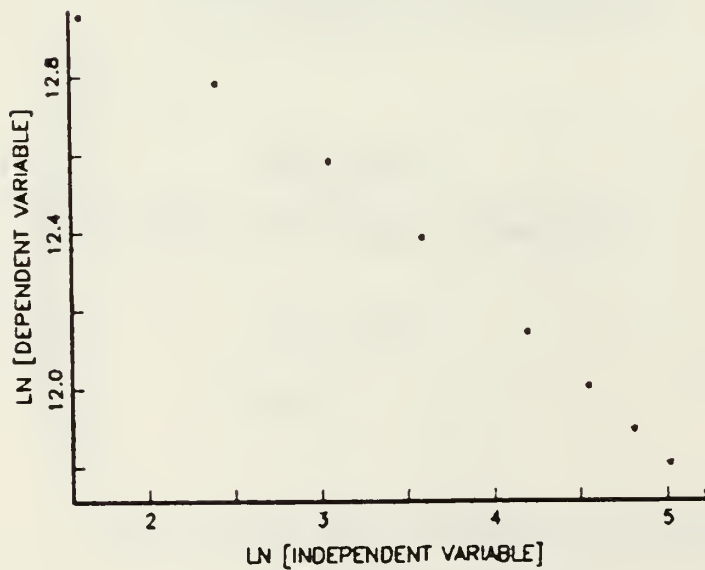


Figure F-1. Cumulative Average Learning Curve: C-141 Data

TABLE F-1

## LINEAR REGRESSION RESULTS

|                                       | $\ln \beta_0$ | (t-stat) | $\beta_1$ | (t-stat) | D.W. | $\hat{\rho}$ | N | $R^2$ | $R^2$ adj |
|---------------------------------------|---------------|----------|-----------|----------|------|--------------|---|-------|-----------|
| ln transformed data                   | 13.603        | (257.85) | -.3524    | (-26.72) | .93  | .5132        | 9 | .99   | .99       |
| AR [1] modeled                        | 12.112        | (5.22)   | -2.66     | (-2.497) | 1.6  | .1550        | 9 | .47   | .40       |
| AR [1] modeled,<br>first obs. omitted | 6.819         | (593.94) | -.4372    | (-85.87) | 1.29 | .3100        | 8 | .99   | .99       |

TABLE F-2

## NON-LINEAR REGRESSION RESULTS

|          | $\beta_0$ | $\beta_1$ | D.W. | $\hat{\rho}$ | N | $R^2$ |
|----------|-----------|-----------|------|--------------|---|-------|
| Raw data | 748887.65 | -.329     | 1.09 | .4267        | 9 | .99   |



# APPENDIX G

## BOGER ET AL MODEL: F-102 DATA ANALYSIS RESULTS

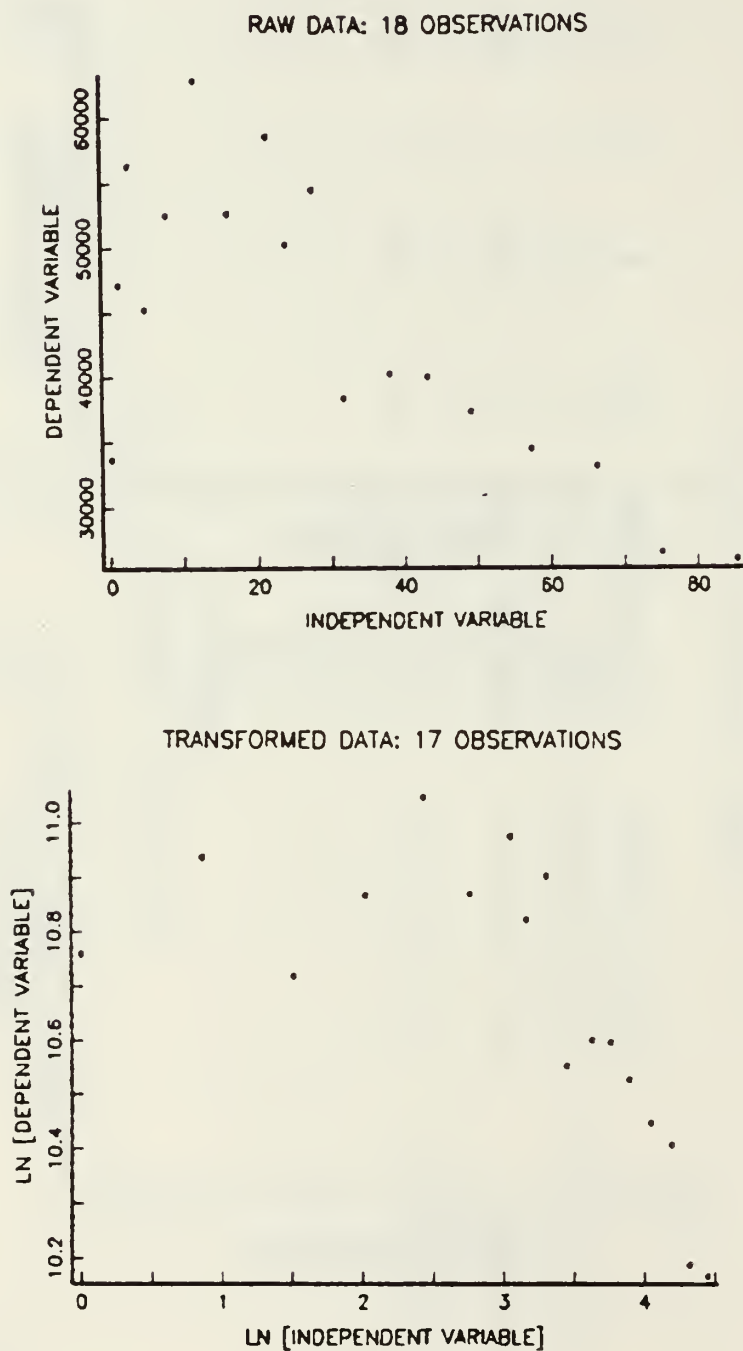


Figure G-1. Boger et al Specification: F-102 Data

TABLE G-1

## LINEAR REGRESSION RESULTS

|                                       | <u>ln <math>\beta_0</math> (t-stat)</u> | <u><math>\beta_1</math> (t-stat)</u> | <u>D.W.</u> | <u><math>\hat{\rho}</math></u> | <u>N</u> | <u>R<sup>2</sup></u> | <u>R<sup>2</sup> adj</u> |
|---------------------------------------|---|--------------------------------------|-------------|--------------------------------|----------|----------------------|--------------------------|
| ln transformed data                   | 11.075 (82.18)                          | -.1349 (-3.24)                       | .66         | .6652                          | 17       | .41                  | .37                      |
| AR [1] modeled                        | 6.893 (14.682)                          | -2.62 (-6.86)                        | 1.13        | .432                           | 17       | .76                  | .74                      |
| AR [1] modeled,<br>first obs. omitted | 4.163 (19.50)                           | -.4935 (-2.93)                       | 2.93        | .489                           | 16       | .38                  | .33                      |

TABLE G-2

## NON-LINEAR REGRESSION RESULTS

|                | <u><math>\beta_0</math></u> | <u><math>\beta_1</math></u> | <u>D.W.</u> | <u><math>\hat{\rho}</math></u> | <u>N</u> | <u>R<sup>2</sup></u> |
|----------------|-----------------------------|-----------------------------|-------------|--------------------------------|----------|----------------------|
| Raw data       | 59636.58                    | -.1002                      | .773        | .6082                          | 17       | .96                  |
| AR [1] modeled | 33420.27                    | -.2864                      | 2.6         | -.3182                         | 17       | .90                  |

## APPENDIX H

### UNIT LEARNING CURVE: F-102 DATA ANALYSIS RESULTS

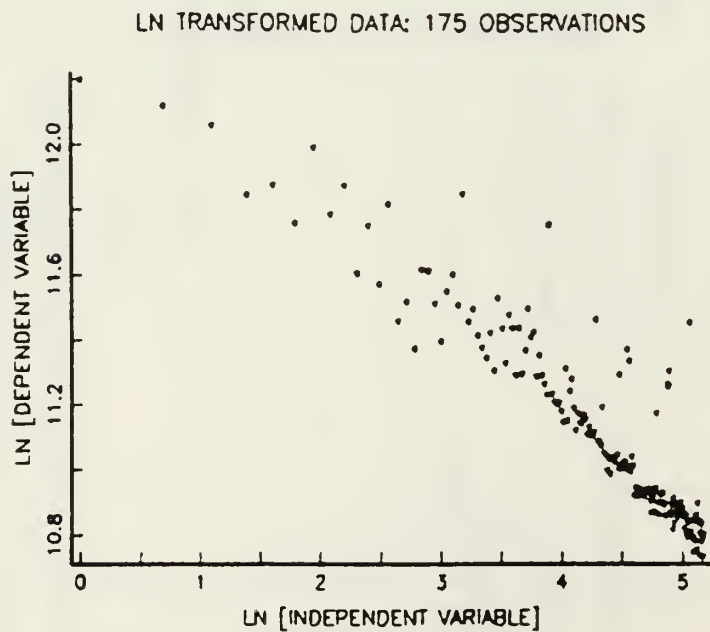
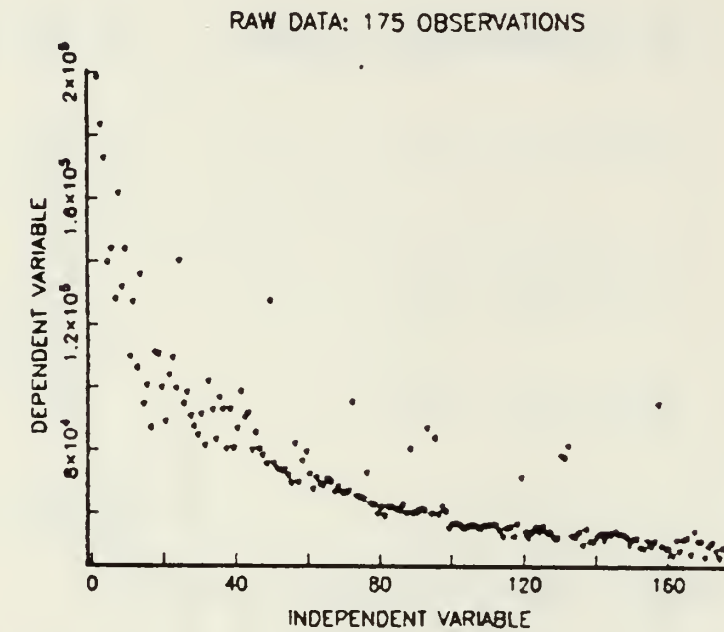


Figure H-1. Unit Learning Curve: F-102 Data

TABLE H-1

## LINEAR REGRESSION RESULTS

|  | <u>ln <math>\beta_0</math></u> | <u>(t-stat)</u> | <u><math>\beta_1</math></u> | <u>(t-stat)</u> | <u>D.W.</u> | <u><math>\hat{\rho}</math></u> | <u>N</u> | <u>R<sup>2</sup></u> | <u>R<sup>2</sup> adj</u> |
|--|--------------------------------|-----------------|-----------------------------|-----------------|-------------|--------------------------------|----------|----------------------|--------------------------|
| ln transformed data,<br>2 outliers omitted | 12.446                         | (351.37)        | -.3120                      | (-37.77)        | 1.55        | .224                           | 173      | .893                 | .892                     |
| AR [1] modeled                             | 9.548                          | (153.34)        | -.400                       | (-20.52)        | 1.45        | .27                            | 173      | .711                 | .709                     |
| AR [1] modeled,<br>first obs. omitted      | 9.687                          | (262.39)        | -.3217                      | (-29.10)        | 2.17        | -.08                           | 172      | .833                 | .832                     |

TABLE H-2

## NON-LINEAR REGRESSION RESULTS

|                                 | <u><math>\beta_0</math></u> | <u><math>\beta_1</math></u> | <u>D.W.</u> | <u><math>\hat{\rho}</math></u> | <u>N</u> | <u>R<sup>2</sup></u> |
|---------------------------------|-----------------------------|-----------------------------|-------------|--------------------------------|----------|----------------------|
| Raw data, 2 outliers<br>omitted | 228139.41                   | -.282                       | 1.65        | .1749                          | 173      | .986                 |

# APPENDIX I

## CUMULATIVE AVERAGE LEARNING CURVE: F-102 DATA ANALYSIS RESULTS

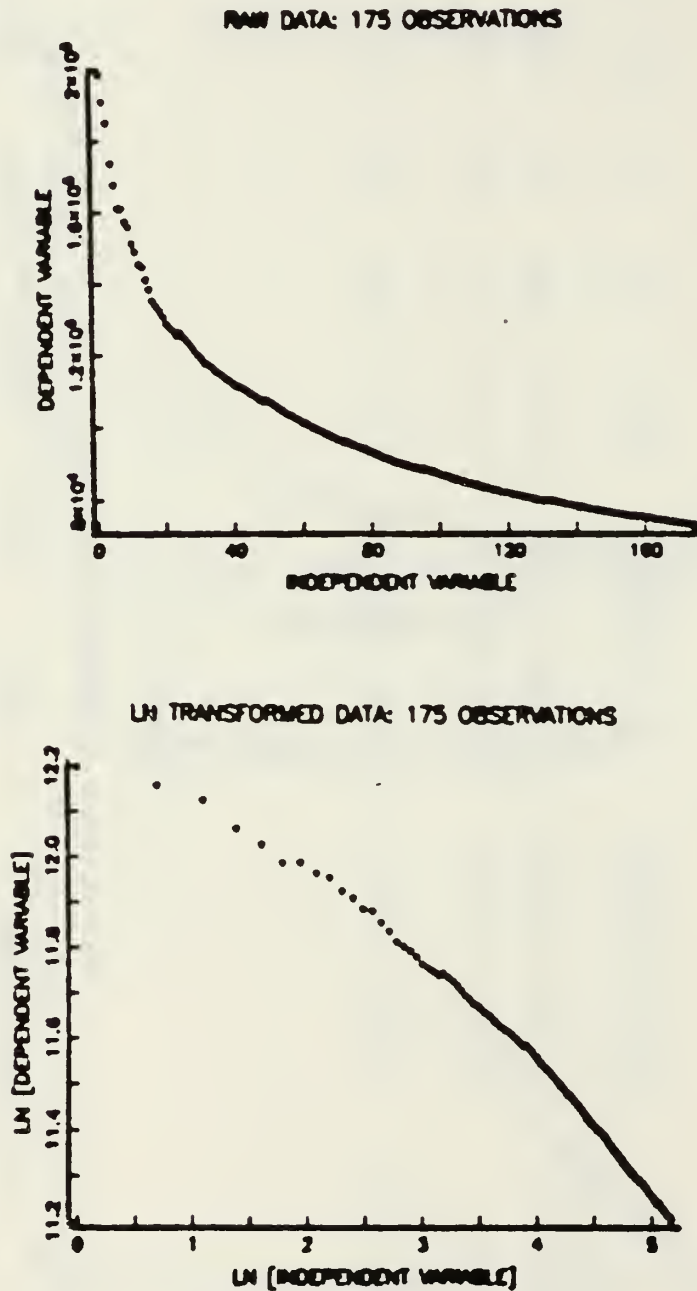


Figure I-1. Cumulative Average Learning Curve: F-102 Data

TABLE I-1

## LINEAR REGRESSION RESULTS

|  | <u><math>\ln \beta_0</math> (t-stat)</u> | <u><math>\beta_1</math> (t-stat)</u> | <u>D.W.</u> | <u><math>\hat{\rho}</math></u> | <u>N</u> | <u>R<sup>2</sup></u> | <u>R<sup>2</sup> adj</u> |
|--|--|--------------------------------------|-------------|--------------------------------|----------|----------------------|--------------------------|
| ln transformed data<br>one outlier omitted | 12.495 (121.97)                          | -.2437 (-102.4)                      | .06         | .9712                          | 174      | .9839                | .9837                    |

TABLE I-2

## NON-LINEAR REGRESSION RESULTS

|                                       | <u><math>\beta_0</math></u> | <u><math>\beta_1</math></u> | <u>D.W.</u> | <u><math>\hat{\rho}</math></u> | <u>N</u> | <u>R<sup>2</sup></u> |
|---------------------------------------|-----------------------------|-----------------------------|-------------|--------------------------------|----------|----------------------|
| Raw data, one outlier<br>omitted      | 250057.20                   | -.227                       | .109        | .9452                          | 174      | .99                  |
| AR [1] modeled                        | 15061.55                    | -.8209                      | 1.0         | .4999                          | 174      | .66                  |
| AR [1] modeled,<br>first obs. omitted | 5913.91                     | -.1434                      | 1.495       | .2524                          | 173      | .97                  |



APPENDIX J  
FITTED MODEL PLOTS

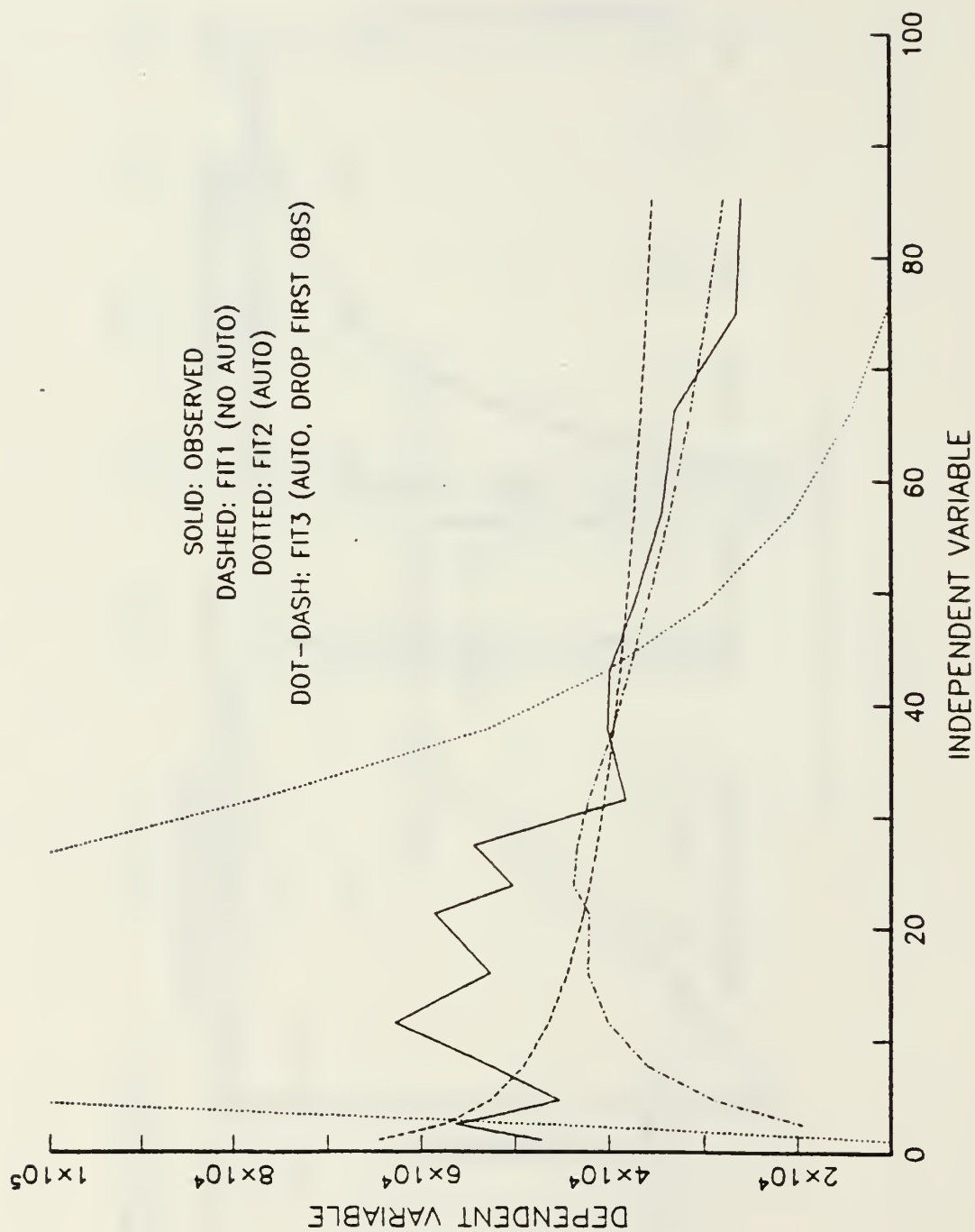


Figure J-1. Linear Regression, Boger et. al. Specification, F-102 Data

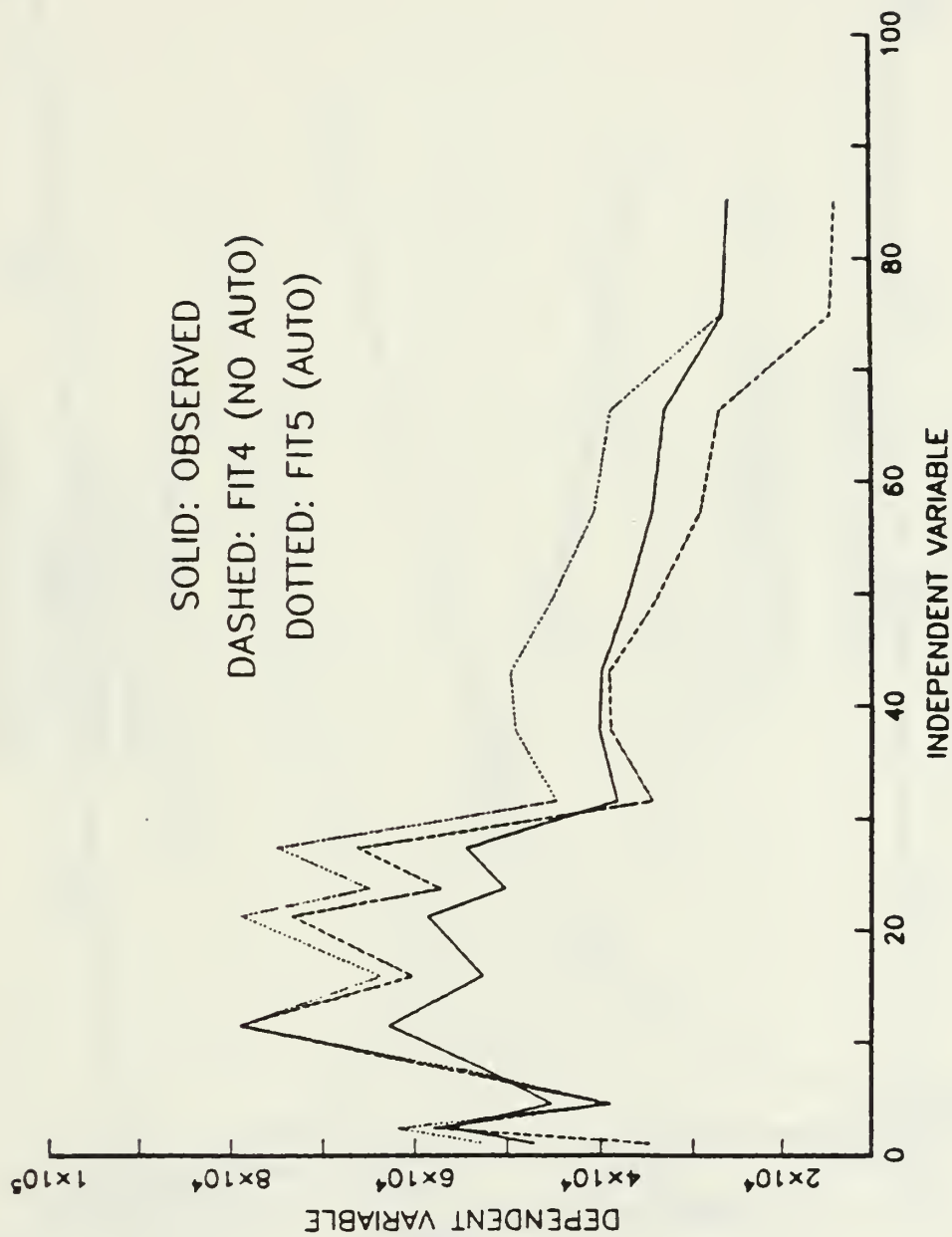


Figure J-2. Non-Linear Regression, Boger et. al. Specification, F-102 Data

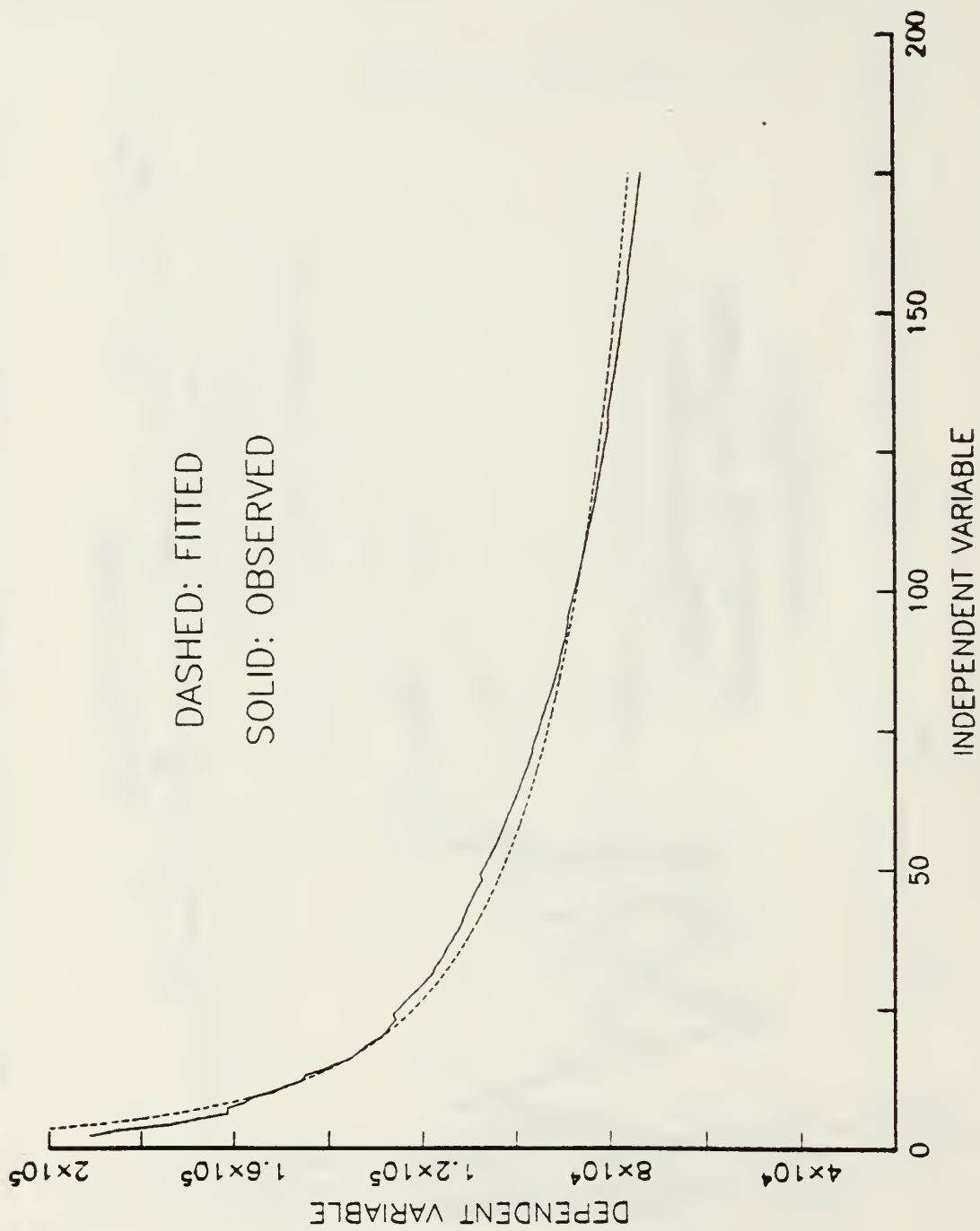


Figure J-3. Linear Regression, Cumulative Average Specification, F-102 Data

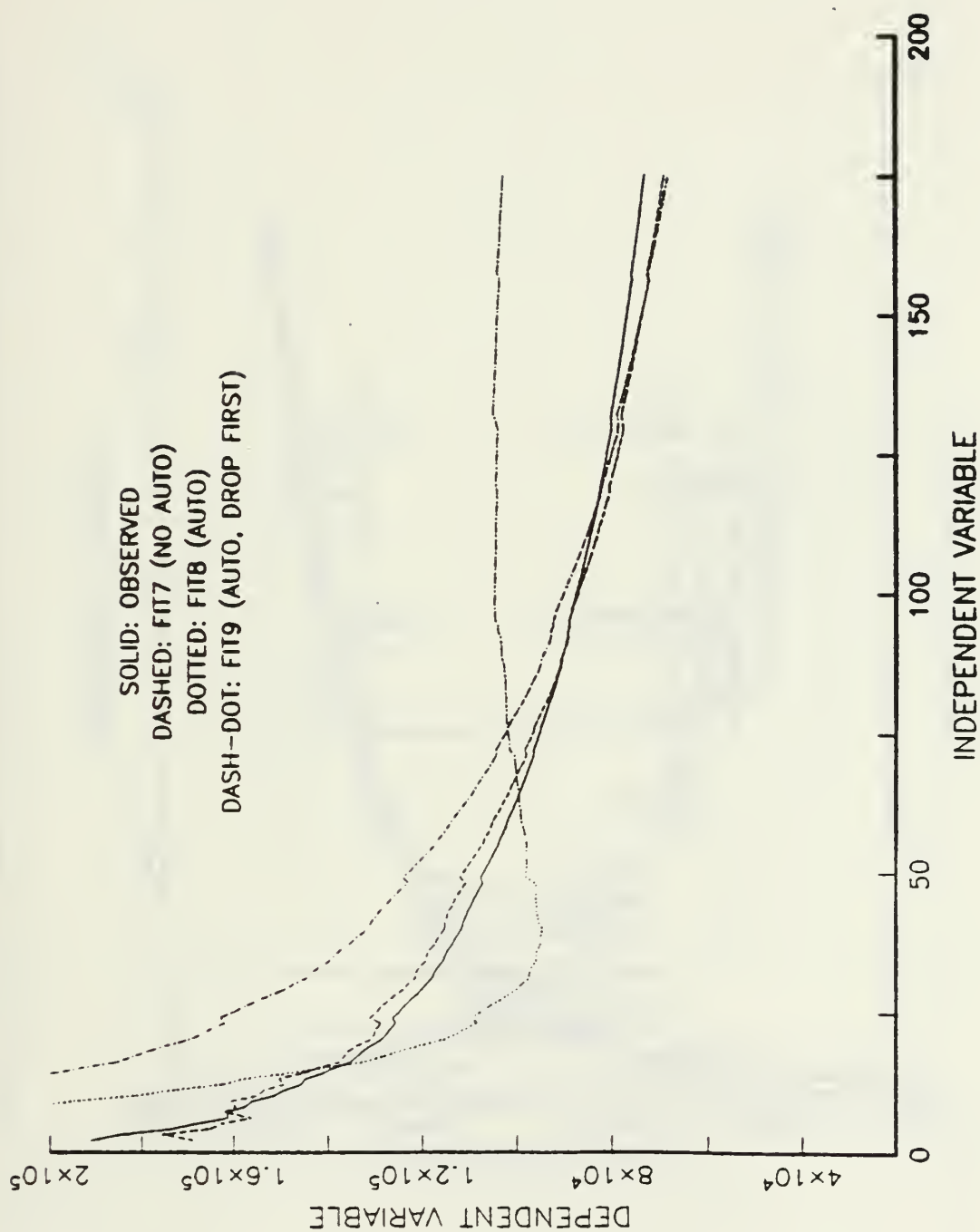


Figure J-4. Non-Linear Regression, Cumulative Average Specification, F-102 Data

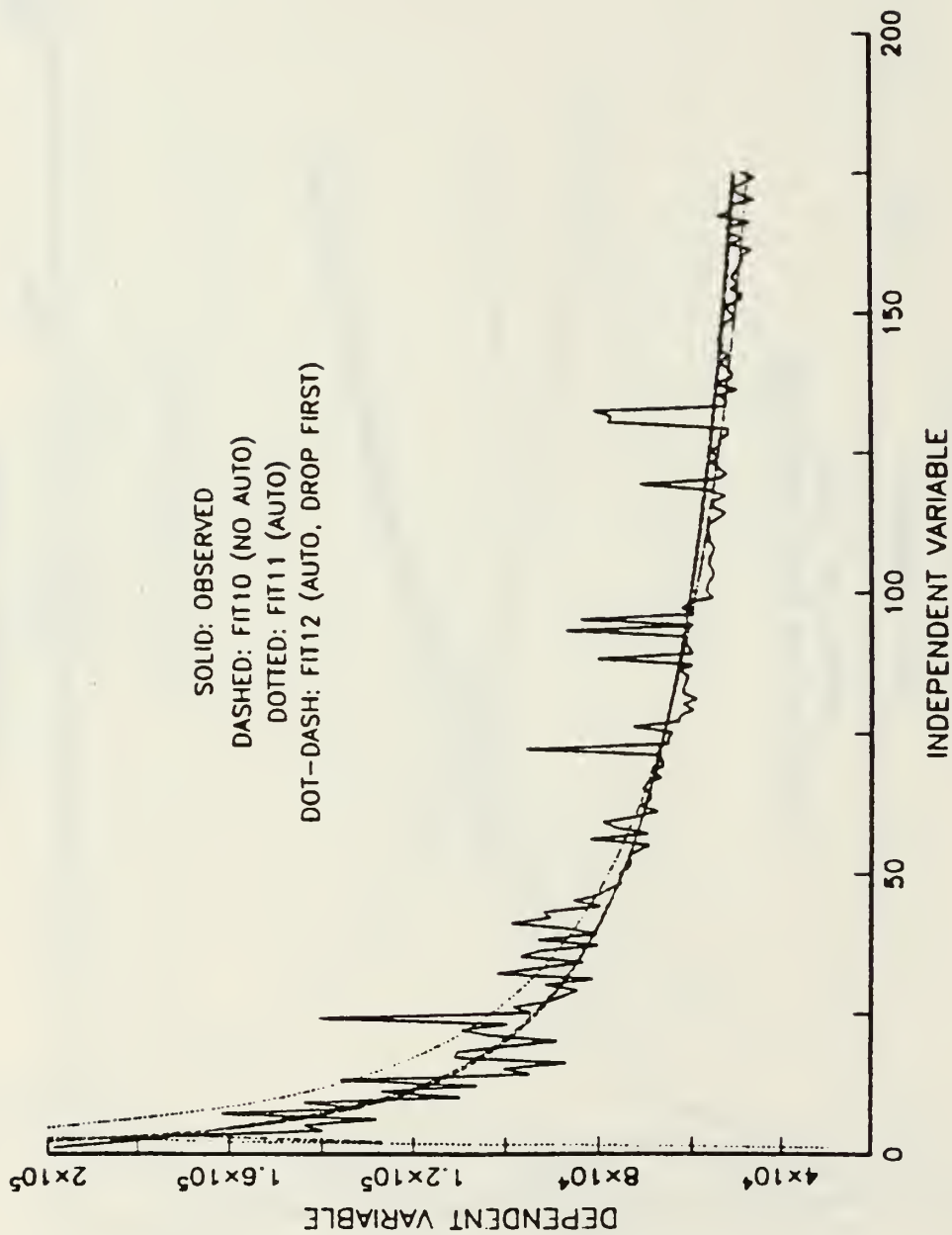


Figure J-5. Linear Regression, Unit Specification, F-102 Data

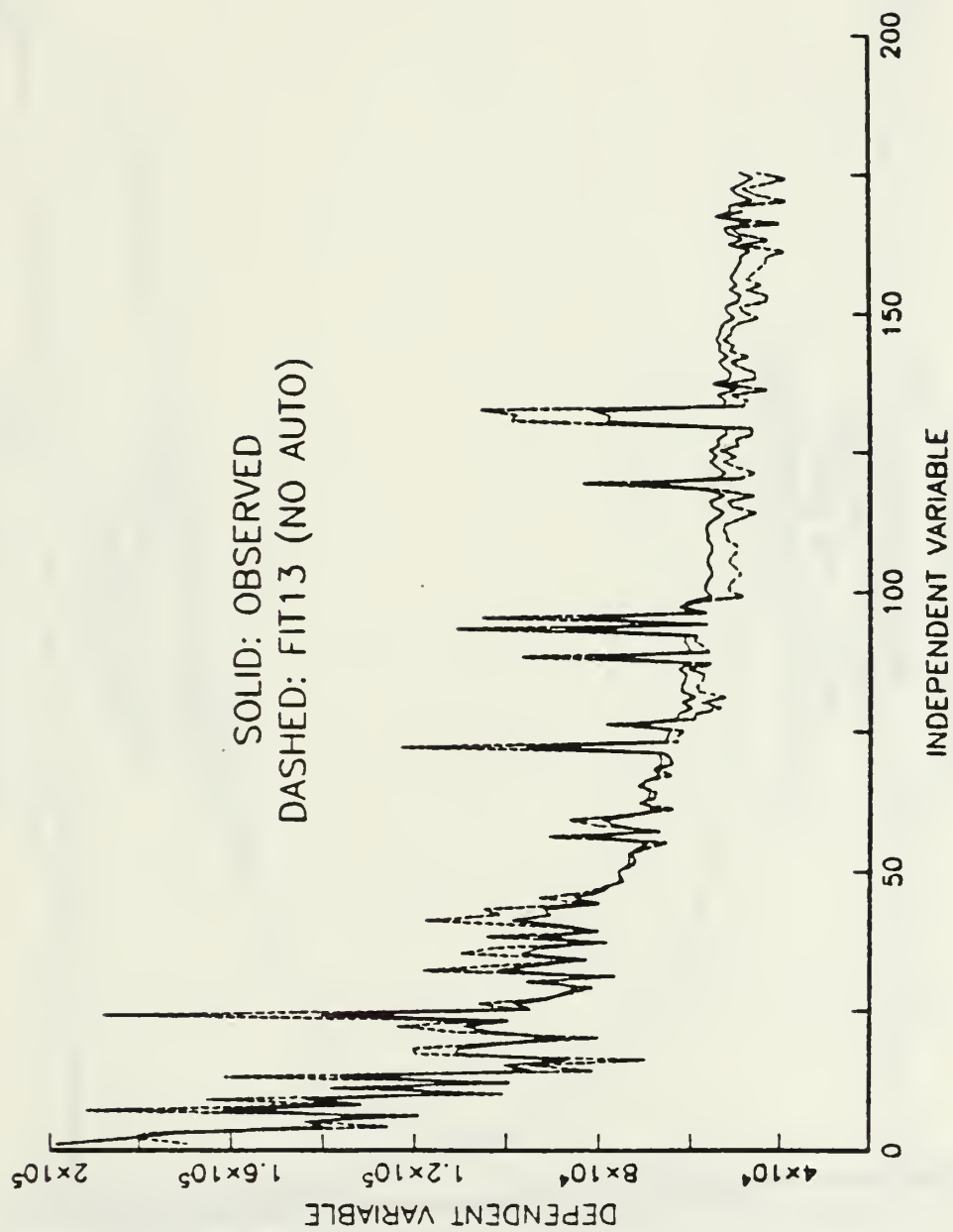


Figure J-6. Non-Linear Regression, Unit Specification, F-102 Data



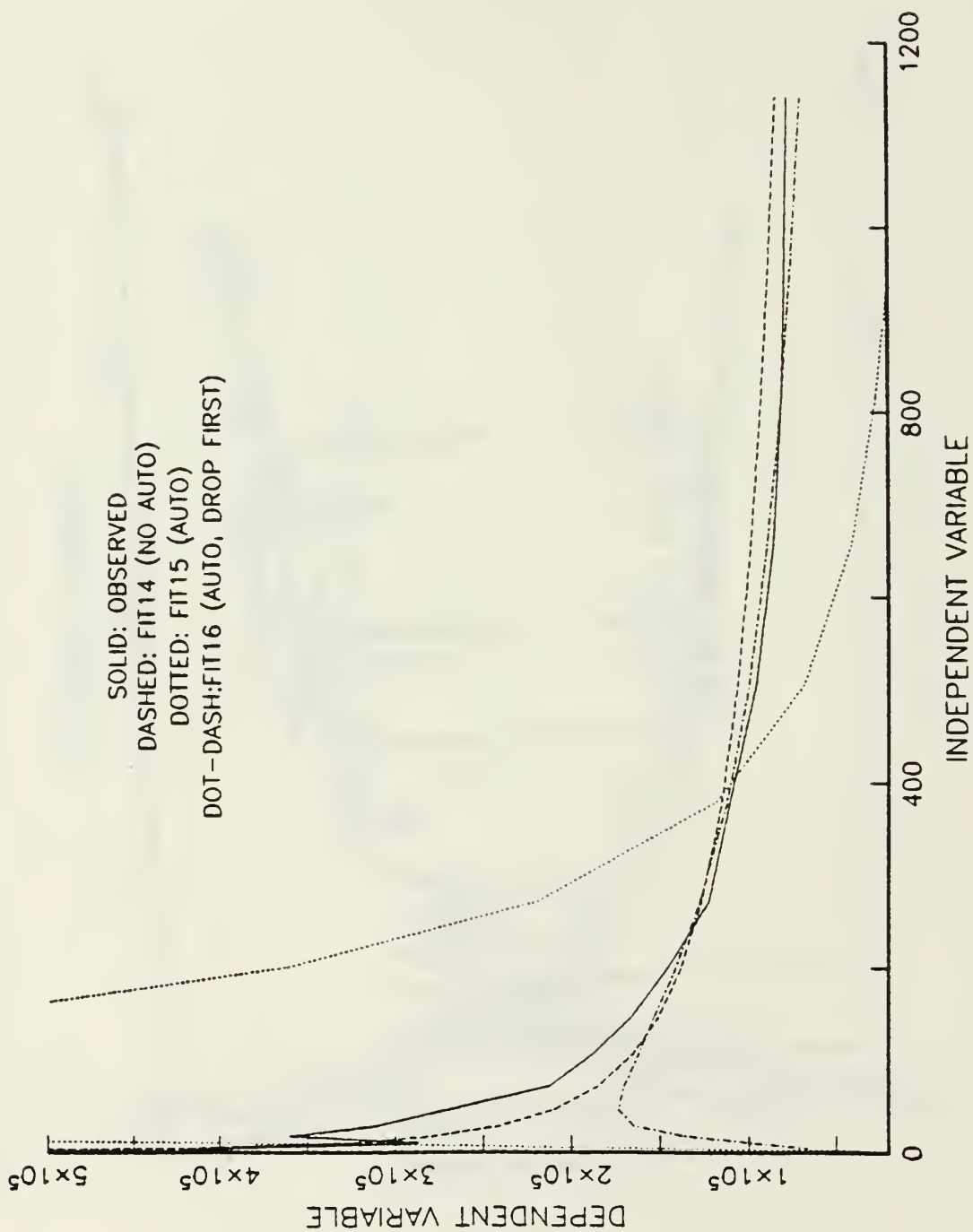


Figure J-7. Linear Regression, Boger et. al. Specification, C-141 Data

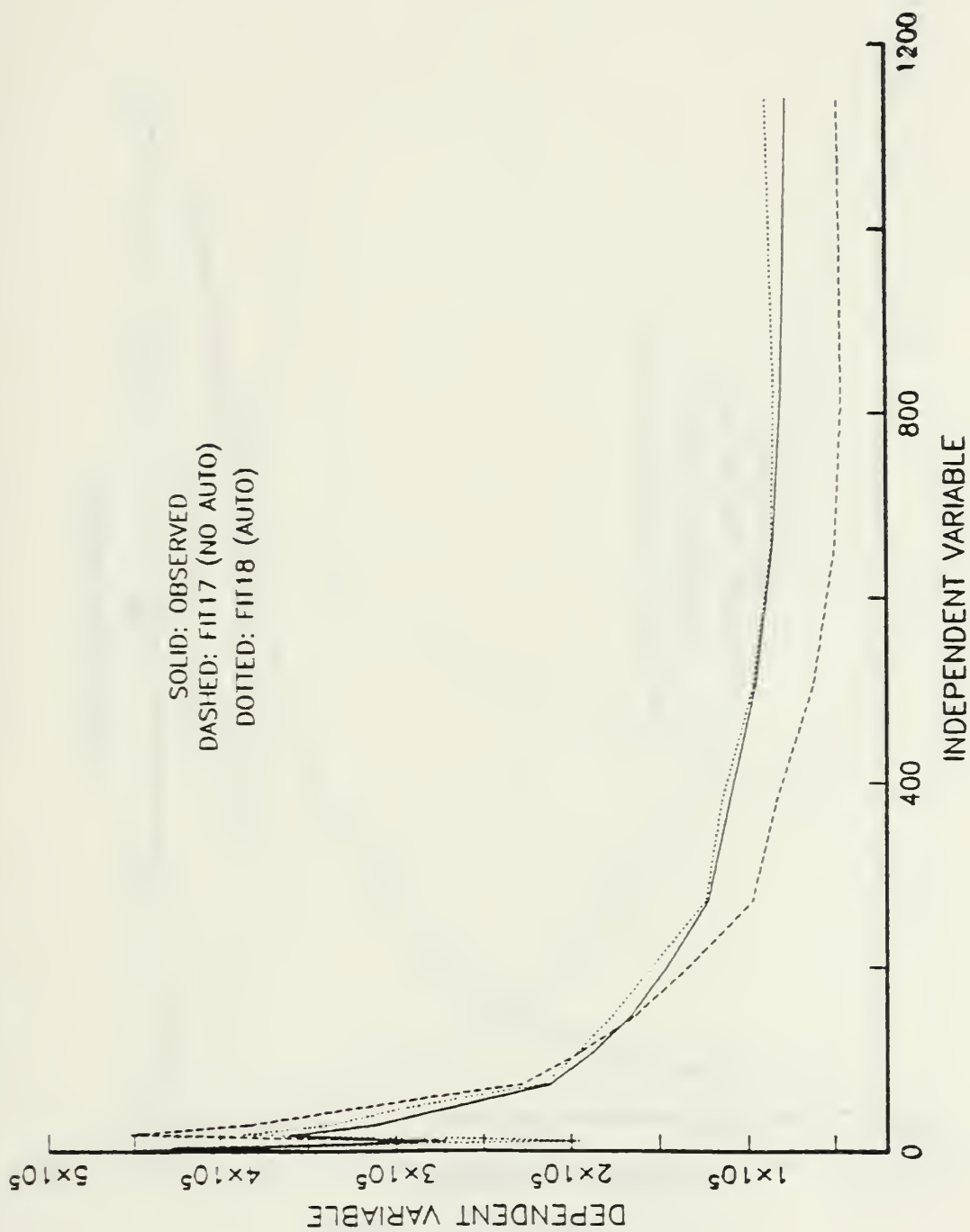


Figure J-8. Non-Linear Regression, Boger et. al. Specification, C-141 Data

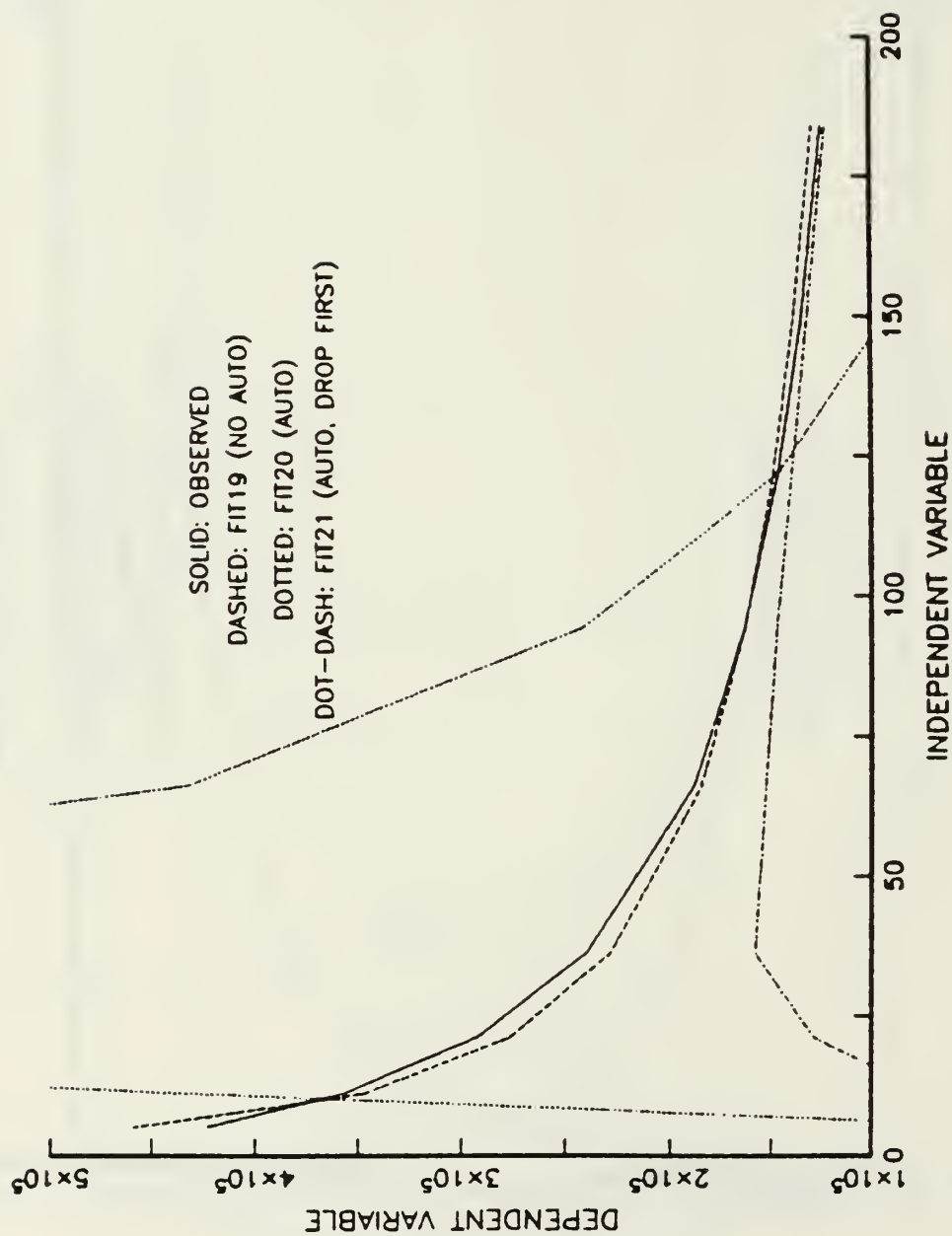


Figure J-9. Linear Regression, Cumulative Average Specification, C-141 Data

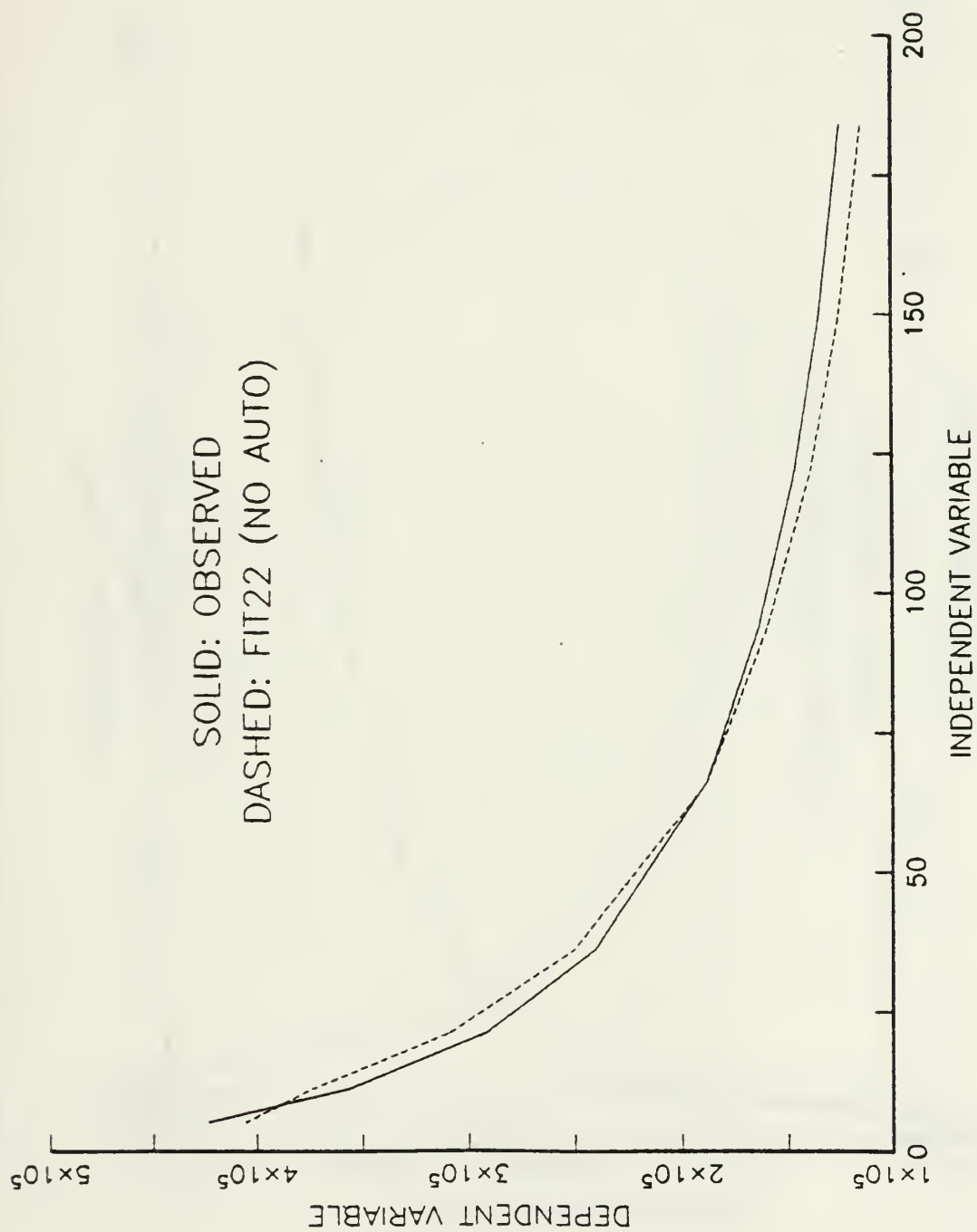


Figure J-10. Non-Linear Regression, Cumulative Average Specification, C-141 Data

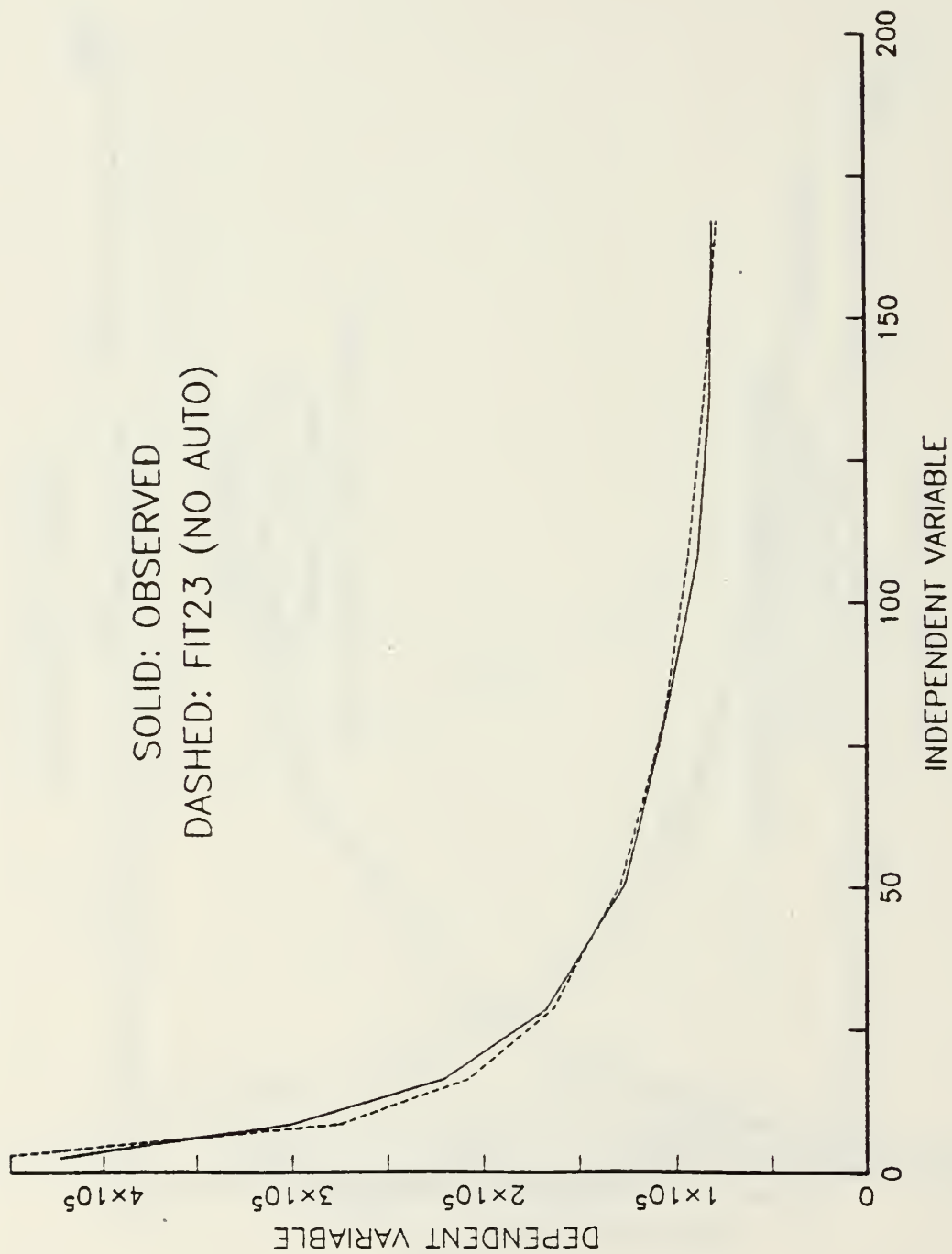


Figure J-11. Linear Regression, Unit Specification, C-141 Data

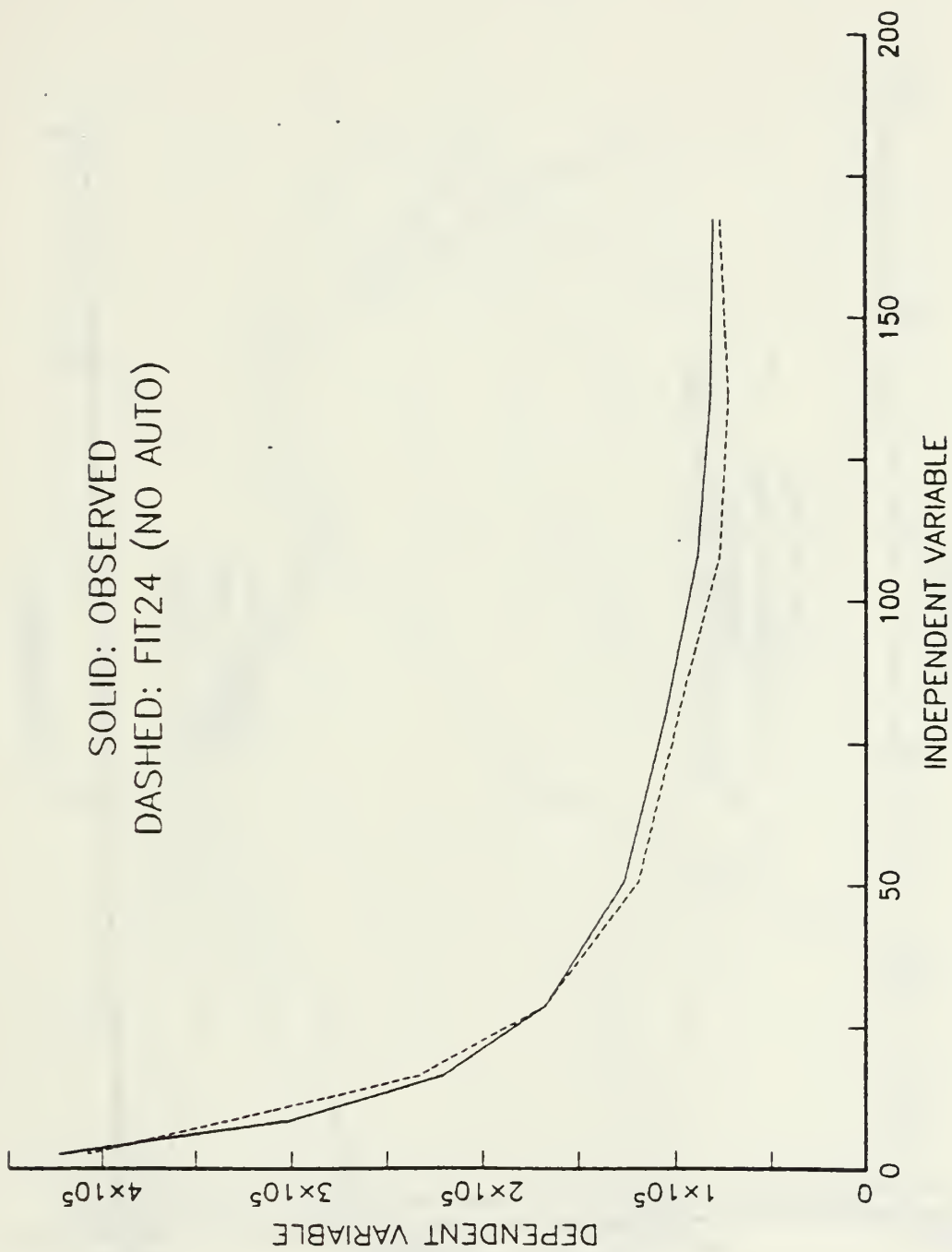


Figure J-12. Non-Linear Regression, Unit Specification, C-141 Data



APPENDIX K  
FITTED LOT COST PLOTS

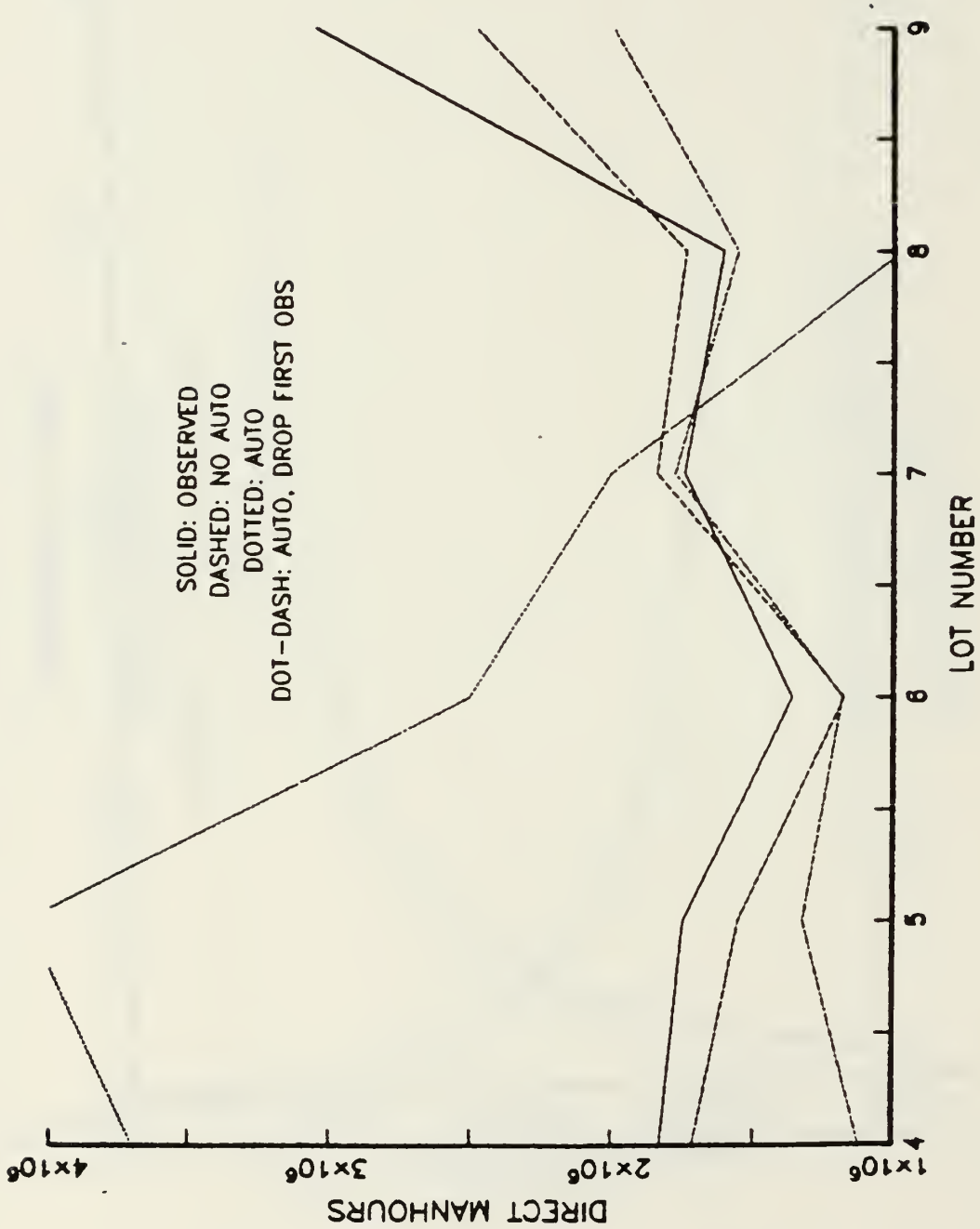


Figure K-1. Boger et. al. Linear Regression Models, F-102 Data

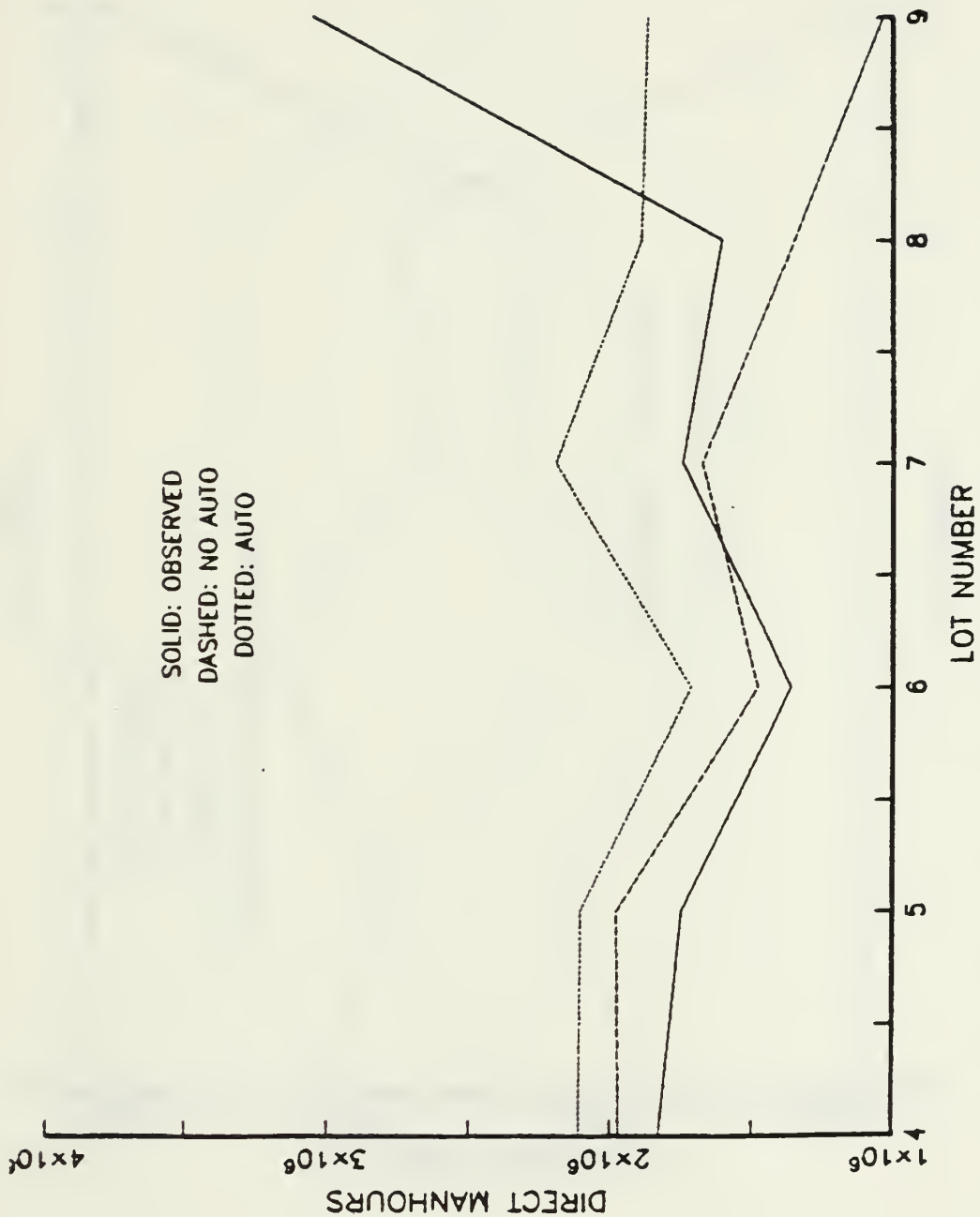


Figure K-2. Boger et. al. Nonlinear Regression Models, F-102 Data

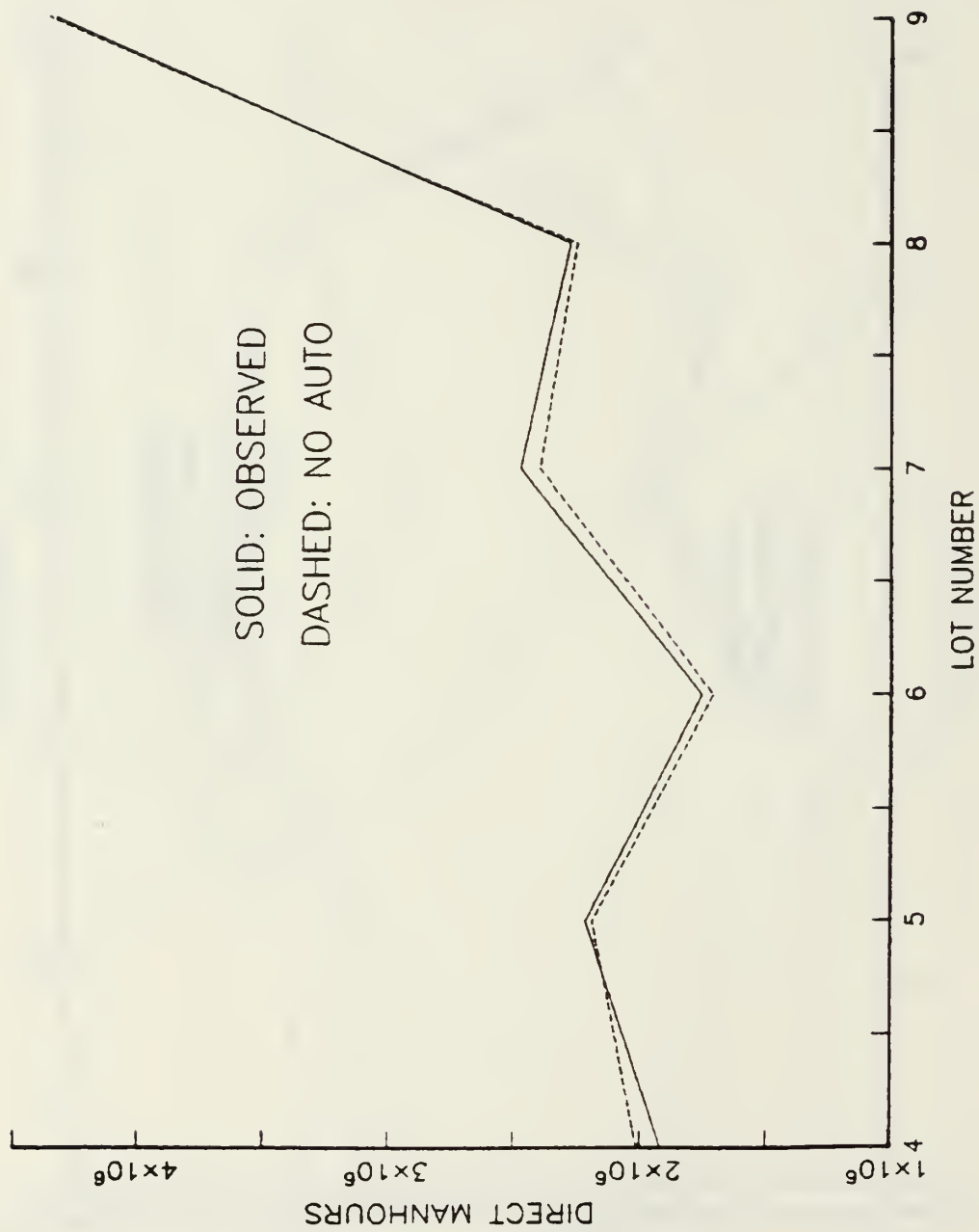


Figure K-3. Cumulative Average Linear Regression Model, F-102 Data

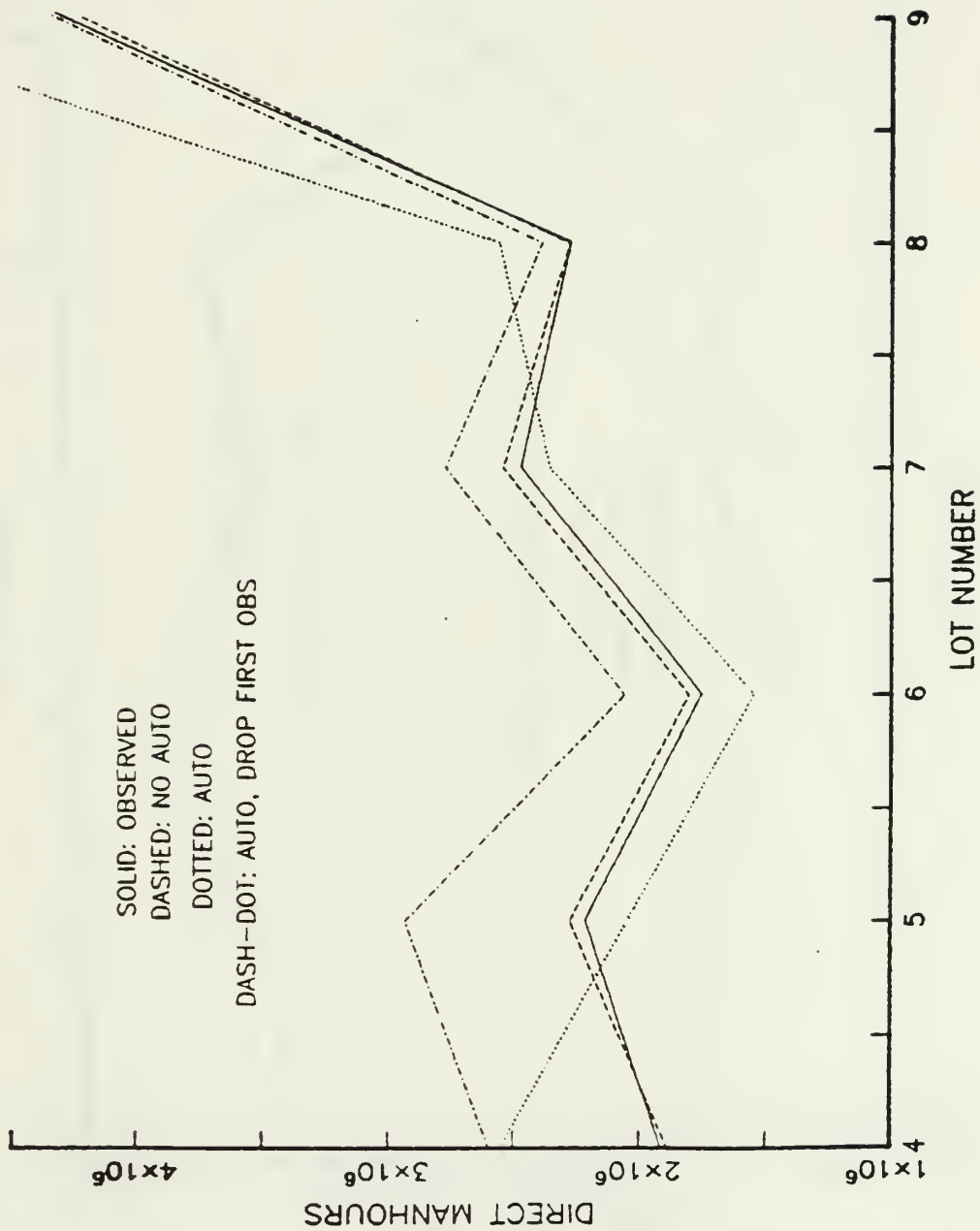


Figure K-4. Cumulative Average Nonlinear Regression Models, F-102 Data

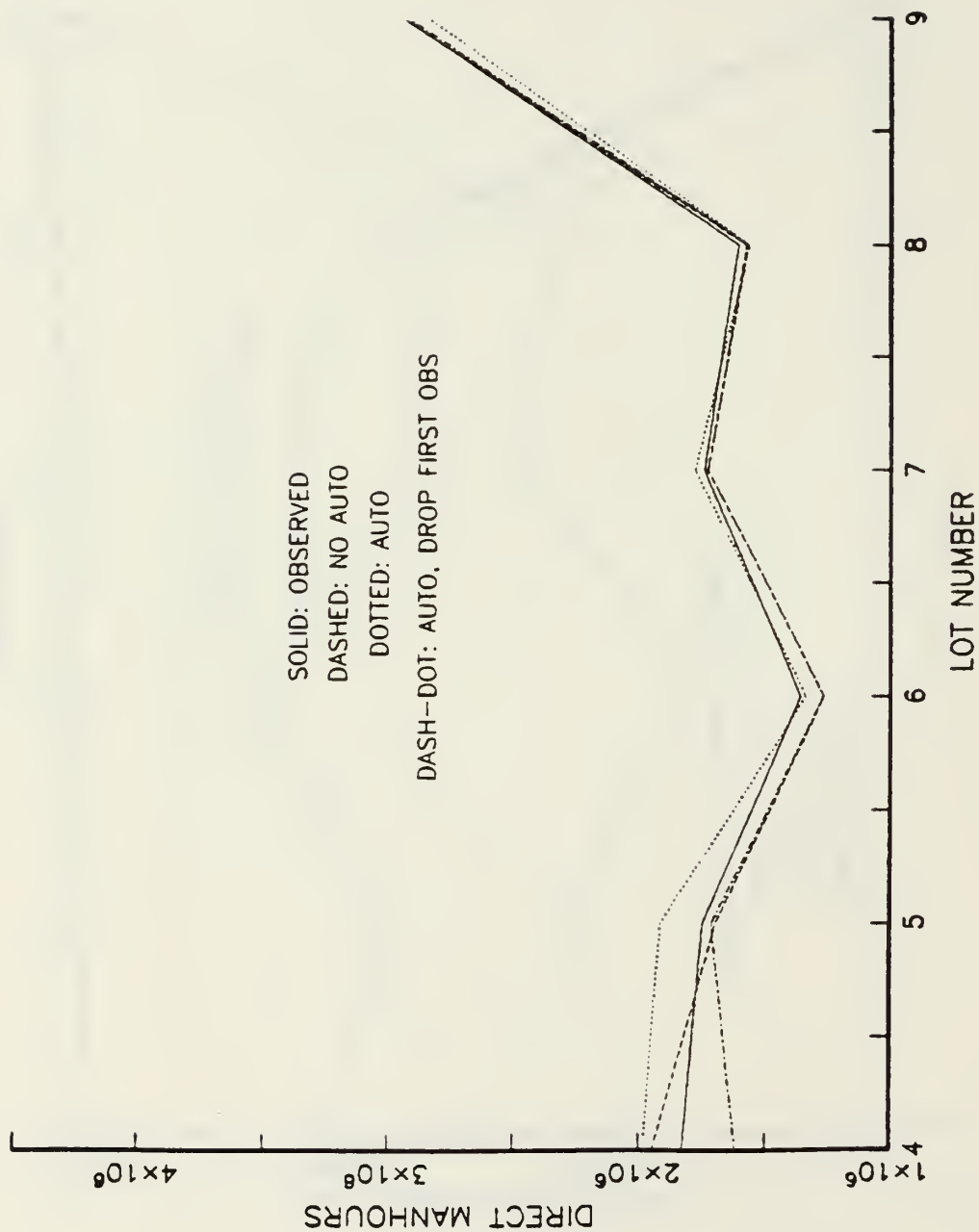


Figure K-5. Unit Linear Regression Models, F-102 Data

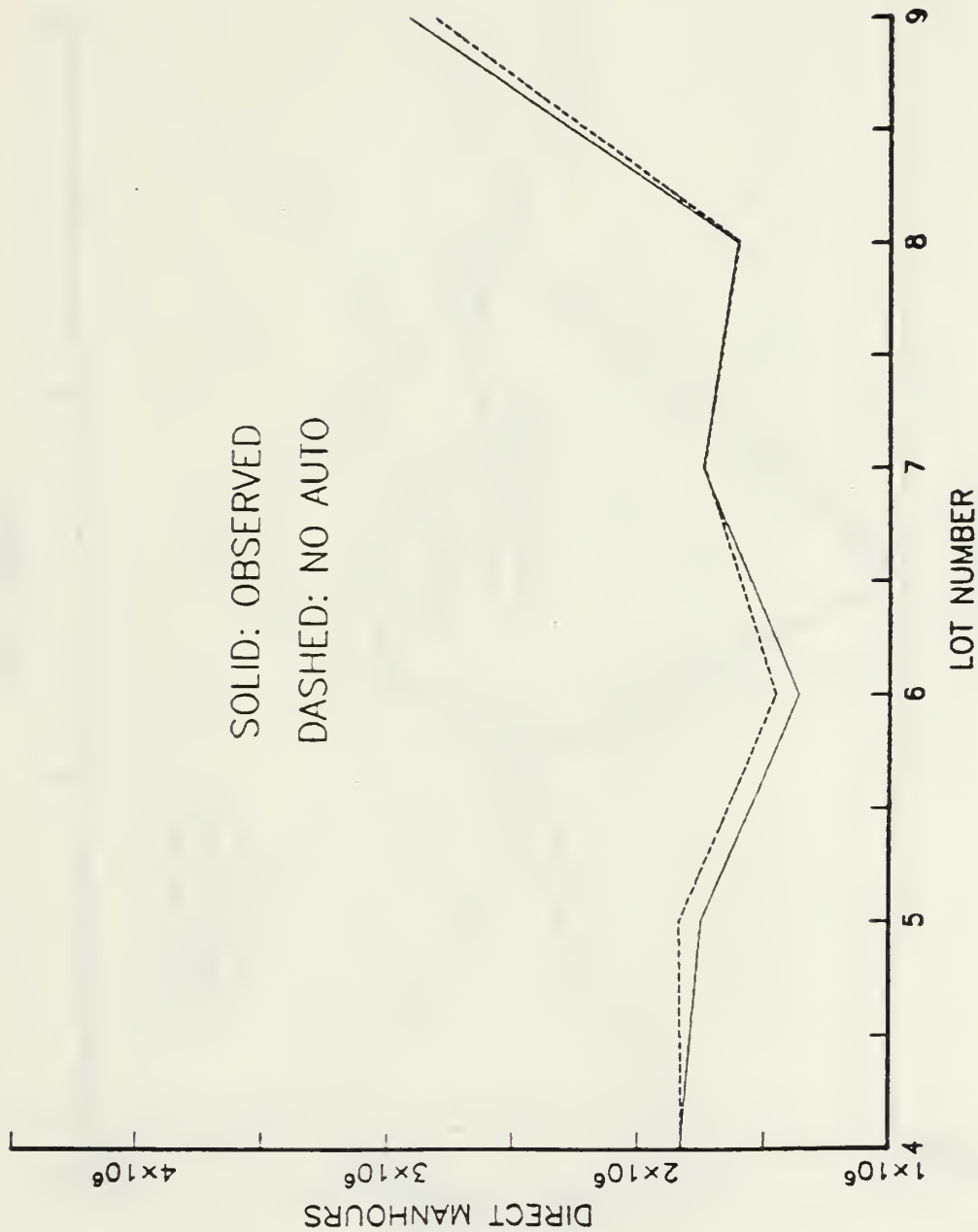


Figure K-6. Unit Nonlinear Regression Model, F-102 Data

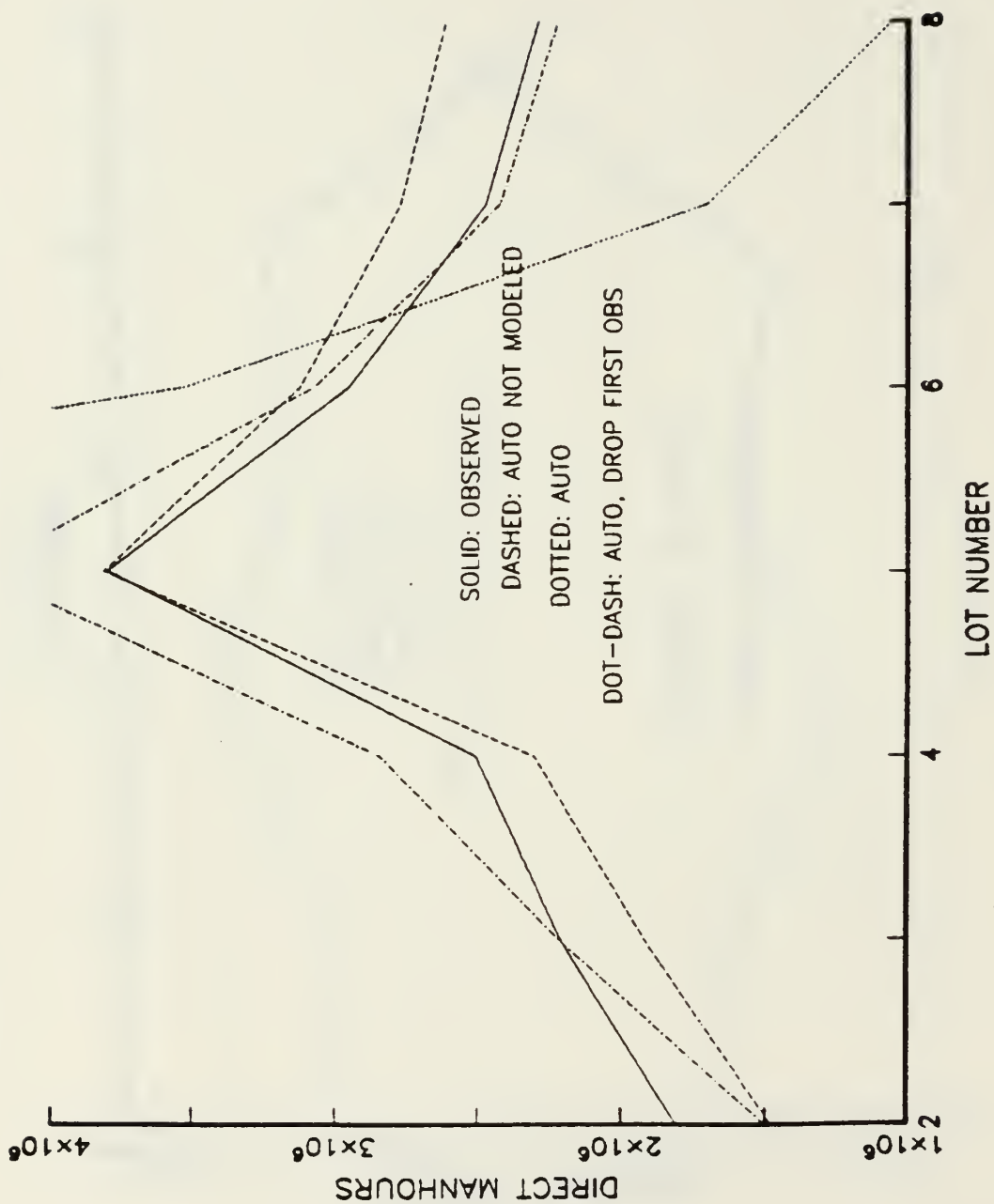


Figure K-7. Boger et. al. Linear Regression Models, C-141 Data



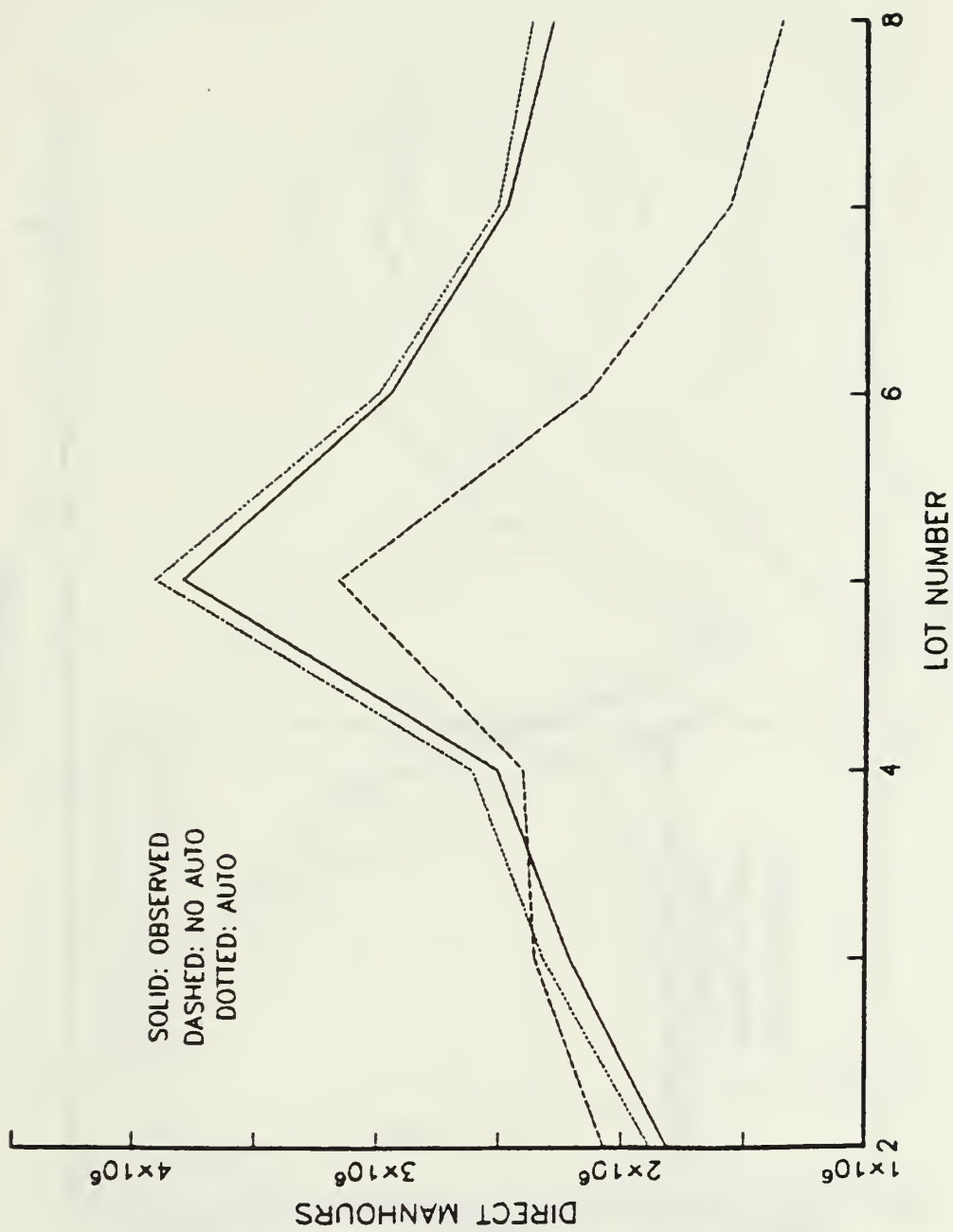


Figure K-8. Boger et. al. Nonlinear Regression Models, C-141 Data

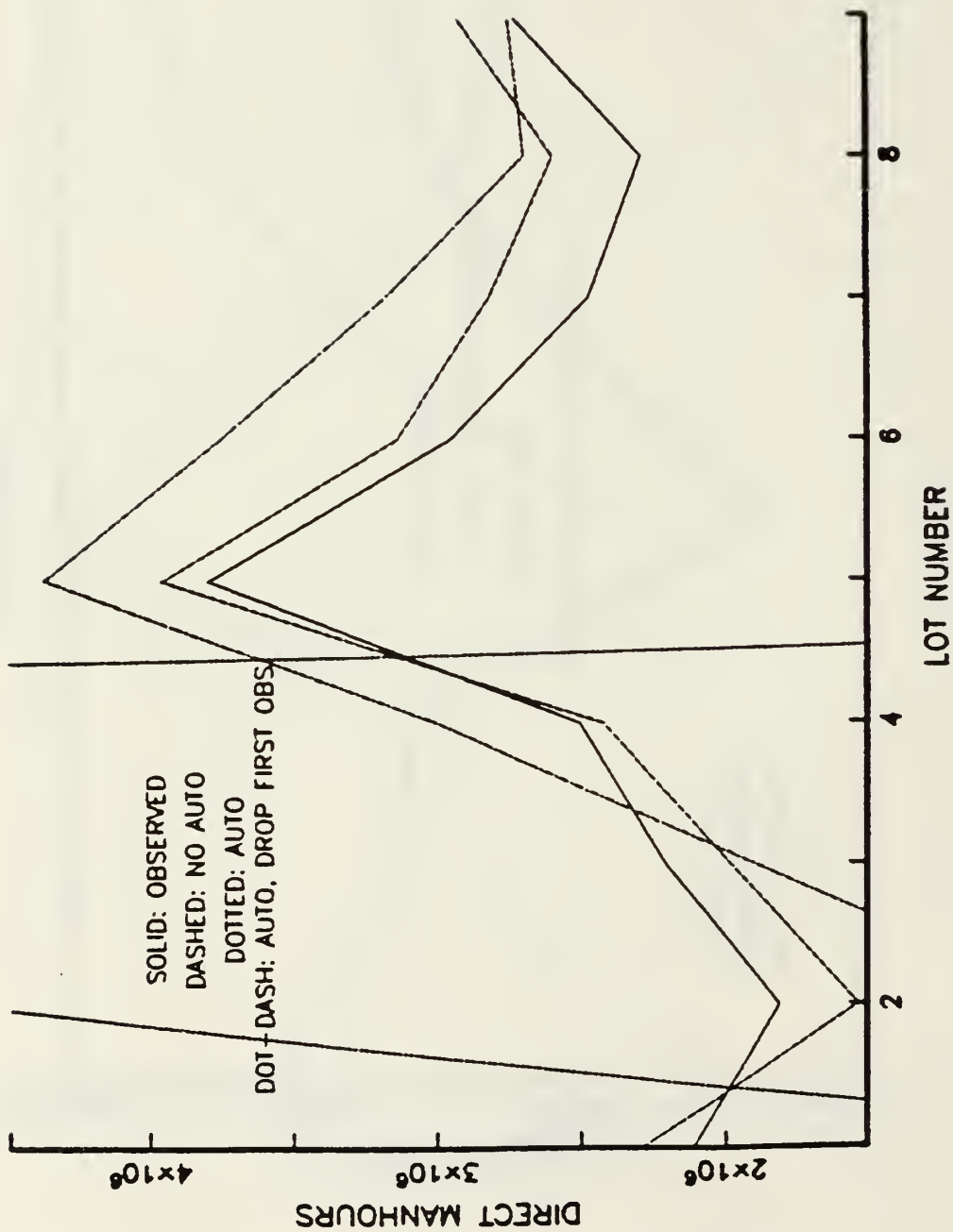


Figure K-9. Cumulative Average Linear Regression Models, C-141 Data

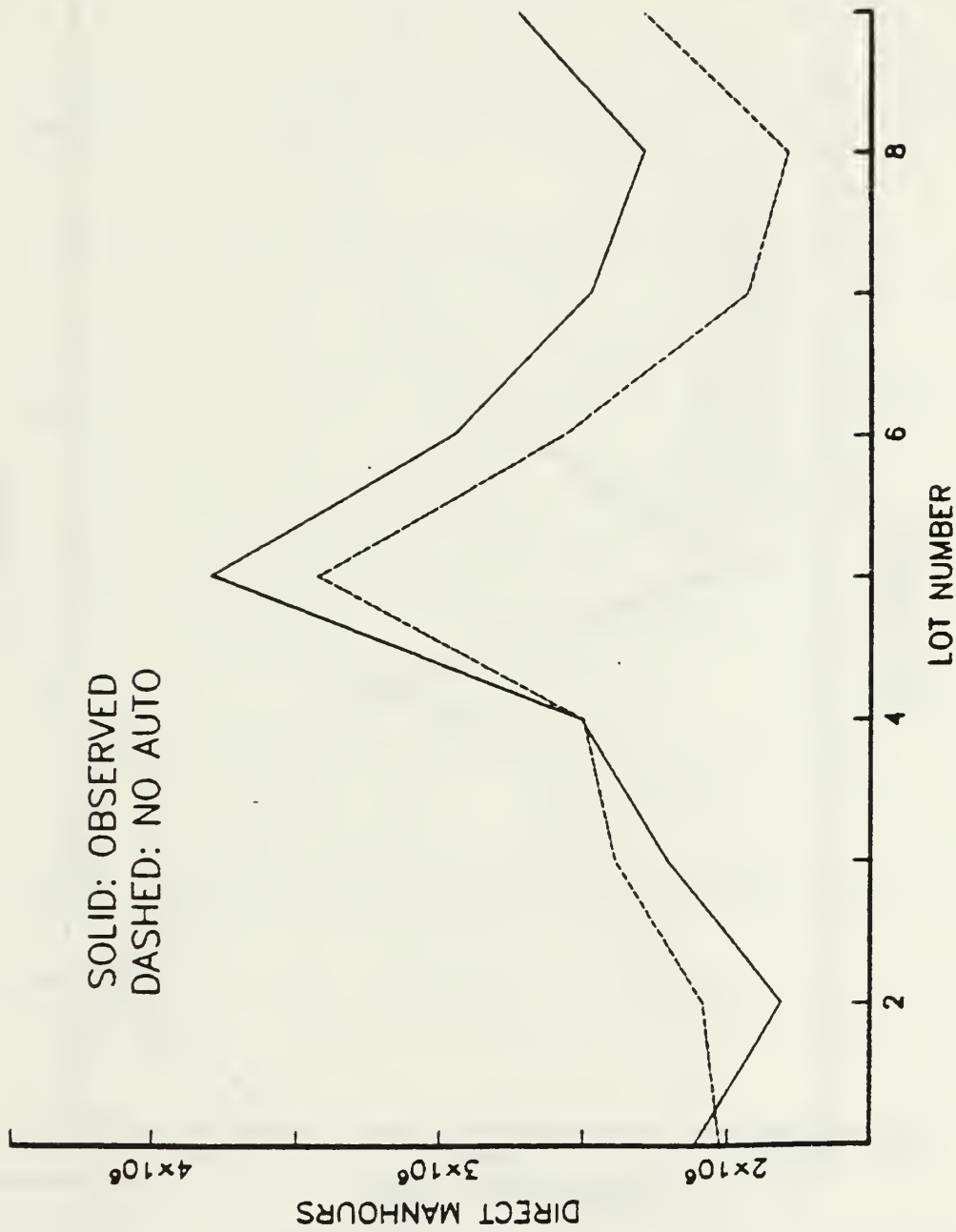


Figure K-10. Cumulative Average Nonlinear Regression Model, C-141 Data

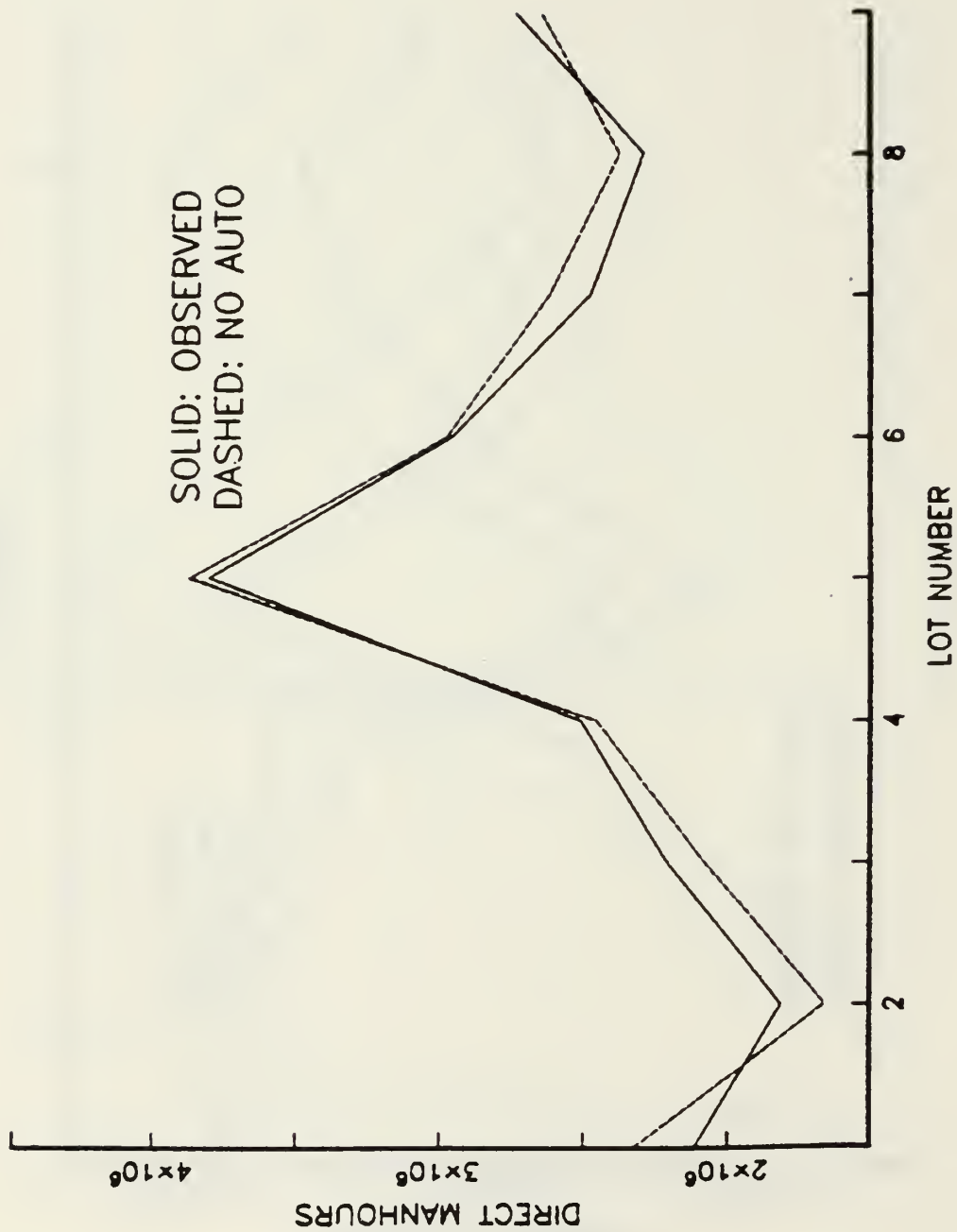


Figure K-11. Unit Linear Regression Model, C-141 Data

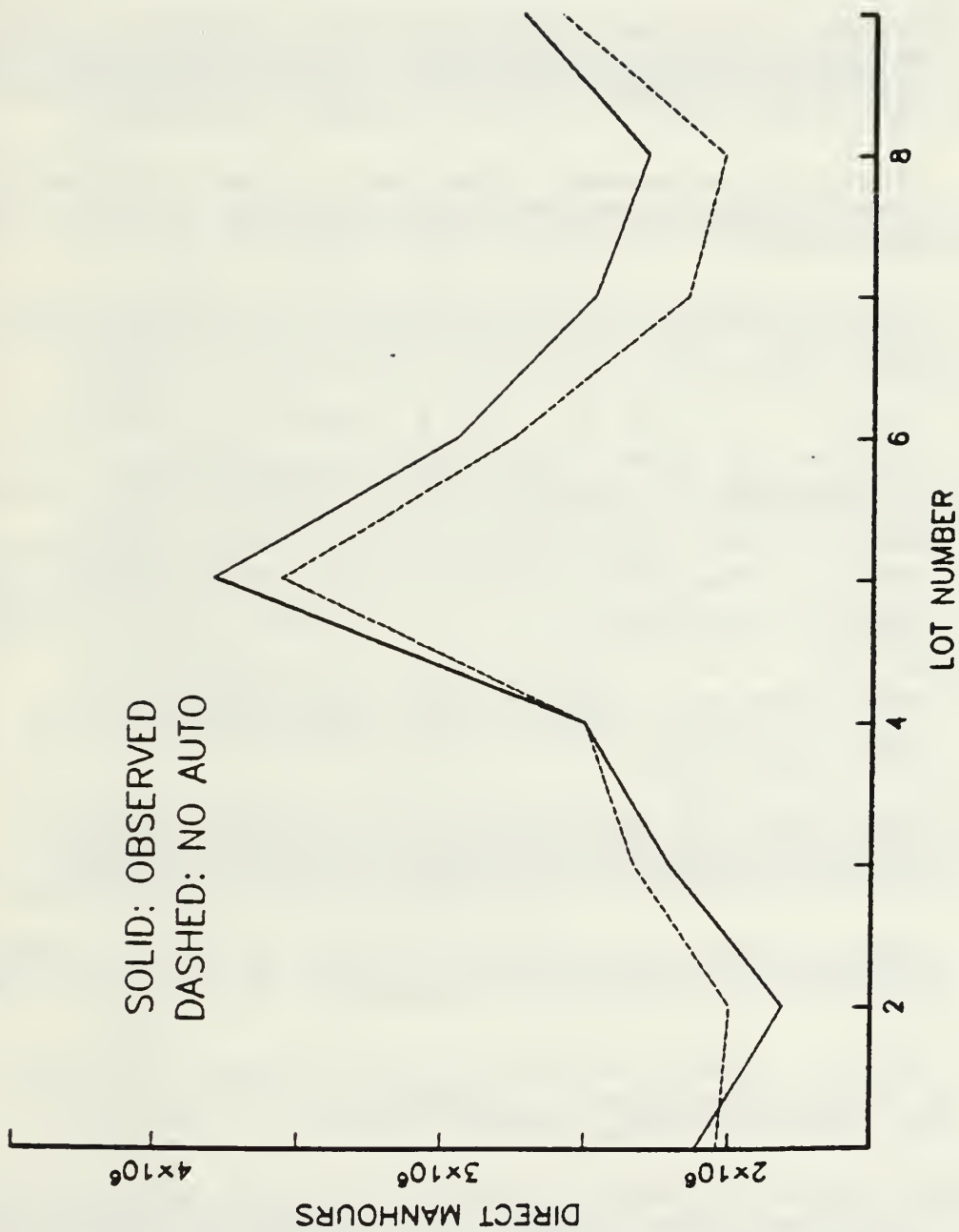


Figure K-12. Unit Nonlinear Regression Model, C-141 Data

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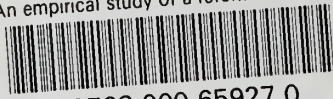
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